

18INIAT11

# First Semester B.F. Degree Examination,,June/Jul:s 2019 Calculus and Linear Algebra 

Time: 3 hrs.
Max. Marks: 100
Note: Answer an;' FIVE full questions, choosing ONE full question from each module.

## Module-1

1 a. With usual notation, prove that $\tan \mathrm{fi}=\mathrm{r} \quad \mathrm{de}^{\prime}$
dr
(06 Marks)
b. Find the radius of curvature of $a^{2} y=-\mathbf{a}^{3}$ at the point where the curve cuts the $x$-axis.
(06 Marks)
C. Show that the evolute of the parabola $y^{2}=4 a x$ is $27 a y^{2}=4\left(x-20^{3}\right.$.
(08 Marks)
OR
2 a . Prove that the pedal equation of the curve $\mathrm{r}^{\prime \prime}=\mathrm{a}{ }^{\prime \prime} \operatorname{cosnO}$ is $\mathrm{a}^{\mathrm{n}} . \mathrm{p}=\mathrm{r}^{n+1}$.
(06 Marks)
b. Show that for the curve $r(-\cos 0)=2 a, p^{2}$ varies as $C$.
(06 Marks)
C. Find the angle between the polar curves $r=a(\quad \cos ())$ and $r=\quad+\cos 0)$.
(08 Marks)

## Module-2

3 a. Expand $\log (\mathrm{I}+\cos \mathrm{x})$ by Maclaurin's series up to the term containing $\mathrm{x}^{4}$.
(06 Nlarks)
b. Evaluate $\begin{array}{cc}\text { lira } & +\mathrm{b}^{\prime}+\mathrm{c}^{\prime} \\ \mathrm{x}-\mathbf{O 1} & 3\end{array}$
(07 Marks)
c. Find the extreme values of the function $\operatorname{tax}, \mathrm{y})=\mathrm{x}^{3}+\mathrm{y}^{3}-3 x-12 \mathrm{y}+20$.
(07 Marks)

## OR

4 a. If $u=f \quad \mathrm{xyz}$ then prove that $\mathrm{x} \bullet-+\mathrm{y}^{\text {"eu }}+\mathrm{z} \cdot--=0$
(06 Marks)

$$
y_{z} x_{j}
$$

b. If $u x+3 y^{-}$v $4 x^{-} y z, w=2 z^{-}-x y$. Evaluate $\left.i\right)(x, y, z)$ at the point $(1,-I, 0)$.
(07 Marks)
c. A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box, if the total surface area is minimum.
(07 Marks)

## Module-3

5 a. Evaluate by changing the order of integration
ax
$x d y-d x a>0$
(06 Marks)
h. Find the area bounded between the circle $x^{-}+y^{-}=a^{*}$ and the line $x+y=a$.
(07 Marks)
c. Prove that $13\left(\mathrm{~m}, \quad \begin{array}{l}\mathrm{m} . \mathrm{n} \\ \hline \quad \mathrm{n}\end{array}\right.$
(07 Marks)

## OR

6 a. Evaluate $1 \mathrm{ff}\left(\mathrm{x}^{2}+\quad+\right)$ dz.dy.dx
(06 Marks)
b. Find the area bounded by the ellipse - $\overline{\mathrm{a}}-\mathrm{-}-1$ by double integration.
(07 Marks)
c. Show that

$$
\begin{gathered}
\mathrm{dO} \mathrm{x}^{7} \\
\mathrm{Vsin} 0
\end{gathered} 1-\mathrm{N} / \sin 0 . \mathrm{d} 0
$$

(07 Marks)

## Module-4

7 a. Solve $\left(1+e^{\prime}\right) d x+e^{\prime}$
(06 Marks)

$$
d y=0
$$

Y,
b. If the air is maintained at $30^{\circ} \mathrm{C}$ and the temperature of the body cools from $80^{\circ} \mathrm{C}$ to $60^{\circ} \mathrm{C}$ in 12 minutes_ Find the temperature of the body after 24 minutes.
(07 Marks)
c. Solve $\mathrm{yp}^{2}+(\mathrm{x}-\mathrm{y}) \mathrm{p}-\mathrm{x}=\mathbf{0}$.
(07 Marks)

## OR

8 a. Solve $\frac{d y}{d x}+y \cdot \tan x=y \cdot \sec x$
(06 Marks)
b. Find the orthogonal trajectory of the family of the curves $r^{n} \bullet e o s n 0=a^{n}$, where $a$ is a parameter.
(07 Marks)
c. Solve the equation $\left(\begin{array}{ll}p x & y\end{array}\right) \cdot(p y+x)=2 p$ byreducing into Clairaut's form taking the substitution $\mathrm{X}=\mathrm{x}^{-}, \mathrm{Y}=$
(07 Marks)

## Modúle-5

9 a_ Find the rank of the matrix

$$
\begin{array}{cccc}
1 & 2 & -2 & 3 \\
5 & -4 & 6 \\
-1 & -3 & 2 & -2 \\
2 & 4 & -1 & 6
\end{array} \quad \text { by applying elementary Row transformations. }
$$

b. Solve the following system of equations by Gauss-Jordan method:
$x+y+z 9,2 x+y-z=0,2 x+5 y+7 z=52$
(07 Marks)
c. Using Rayleigh's power method find the largest eigen value and corresponding eigen vecto 01
of the matrix $A=020$ with $X^{(' " ~}=(\mathbf{1}, \mathbf{0}, 0)^{\prime \prime}$ as the initial eigen vector carry out O 2

5 iterations.
(07 Marks)

## OR

10 a. For what values of $r$. and $\mu$ the system of equations.
$x+y+z=6, x+2 y+3 z---10, x+2 y+x y$. p may have
i) Unique solution
ii) Infinite number of solutions
n ) No solution.
(06 Marks)
h. Reduce the matrix $\left.\mathrm{A}=\left|\begin{array}{rr}-1 & \\ 2 & 4\end{array}\right| \right\rvert\,$ into diagonal form.
(07 Marks)
c. Solve the following system of equations by Gauss-Seidel method

$$
20 \mathrm{x}+\mathrm{y}-2 \mathrm{z}=17,3 \mathrm{x}+20 \mathrm{y}-\mathrm{z}=-18,2 \mathrm{x} 3 \mathrm{y}+20 \mathrm{z}=25 . \text { Carry out } 3 \text { iterations. } \quad(\mathbf{0 7} \text { Marks) }
$$

