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First Semester B.F. Degree Examination, June/July 2019
Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notation, prove that $\tan \phi = r \frac{de'}{dr}$ (06 Marks)
- b. Find the radius of curvature of $a^2y = -x^3$ at the point where the curve cuts the x-axis. (06 Marks)
- c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 20)^3$. (08 Marks)

OR

- 2 a. Prove that the pedal equation of the curve $r^n = a^n \cos n\theta$ is $a^n \cdot p = r^{n+1}$. (06 Marks)
- b. Show that for the curve $r(-\cos\theta) = 2a$, p^2 varies as C . (06 Marks)
- c. Find the angle between the polar curves $r = a(1 + \cos\theta)$ and $r = a(1 + \sin\theta)$. (08 Marks)

Module-2

- 3 a. Expand $\log(1 + \cos x)$ by Maclaurin's series up to the term containing x^4 . (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x - 3}{x - 0}$. (07 Marks)
- c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. (07 Marks)

OR

- 4 a. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ (06 Marks)
- b. If $u = x^2 + 3y^2 - 4x^2yz, w = 2z^2 - xy$. Evaluate $\frac{\partial^2(u, v, w)}{\partial x \partial y \partial z}$ at the point (1, -1, 0). (07 Marks)
- c. A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box, if the total surface area is minimum. (07 Marks)

Module-3

- 5 a. Evaluate by changing the order of integration $\int_0^a \int_{x^2}^x y \, dy \, dx, a > 0$ (06 Marks)
- b. Find the area bounded between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$. (07 Marks)
- c. Prove that $\int_0^m \int_0^n \frac{1}{x^2 + y^2} \, dx \, dy = \frac{1}{n} \log \frac{m}{n}$ (07 Marks)

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OR

6 a. Evaluate $\int_{-b}^b \int_{-a}^a f(x^2 + y^2) dz dy dx$ (06 Marks)

b. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. (07 Marks)

c. Show that $\frac{dO}{V \sin \theta} x^{\frac{1}{2}} = \frac{1}{1 - N/\sin \theta} d\theta$ (07 Marks)

Module-4

7 a. Solve $(1 + e^y)dx + e^y dy = 0$ (06 Marks)

b. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes. Find the temperature of the body after 24 minutes. (07 Marks)

c. Solve $yp^2 + (x - y)p - x = 0$. (07 Marks)

OR

8 a. Solve $\frac{dy}{dx} + y \cdot \tan x = y \cdot \sec x$ (06 Marks)

b. Find the orthogonal trajectory of the family of the curves $r^n \cdot \cos n\theta = a^n$, where a is a parameter. (07 Marks)

c. Solve the equation $(px - y) \cdot (py + x) = 2p$ by reducing into Clairaut's form taking the substitution $X = x^2$, $Y = y^2$. (07 Marks)

Module-5

9 a. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{pmatrix}$$

by applying elementary Row transformations. (06 Marks)

b. Solve the following system of equations by Gauss-Jordan method:
 $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$ (07 Marks)

c. Using Rayleigh's power method find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}$ with $X^{(1)} = (1, 0)^T$ as the initial eigen vector carry out 5 iterations. (07 Marks)

OR

10 a. For what values of r and μ the system of equations.
 $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = p$ may have
 i) Unique solution ii) Infinite number of solutions iii) No solution. (06 Marks)

b. Reduce the matrix $A = \begin{pmatrix} -1 & 2 \\ 2 & 4 \end{pmatrix}$ into diagonal form. (07 Marks)

c. Solve the following system of equations by Gauss-Seidel method
 $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x + 3y + 20z = 25$. Carry out 3 iterations. (07 Marks)