USN 18IVIAT11

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First Semester B.F. Degree Examination, June/July 2019
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE MI questions, choosing ONE full question from each module.

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Module-I

1 a. With usual notation, prove that tan (1)– r (10 dr . **(06 Marks)**

b. Find the radius of curvature of $a^2y = x^3 - a^3$ at the point where the curve cuts **the** x-axis. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)'$. **(08 Marks)**

OR

2 a. Prove that the pedal equation of the curve r^n = a"cosnO is a^n.p = r"' ^{I}. **(06 Marks)**

b. Show that for the curve r(1 – cos0) = 2a, $\mathbf{p}^2$ varies as r'. **(06 Marks)**

c. Find the angle between the polar curves r = a(1 – cos()) and r = b(1 + cos0). **(08 Marks)**

Module 2

3 a. Expand log(1+ cosx) by Maclaurin's series up to the term containing x^4. **(07 Marks)**

b. Evaluate lim x 01 jax +b` 3 **(07 Marks)**

c. Find the extreme values of the function tax, y) $= x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

OR

4 a. If u f (x y z y z x then prove that x • $\frac{au}{}$ + y ?u + z • $\frac{au}{cz}$ =0 **(06 Marks)**

b. If u = x + $3y^2 - z^3$, v = 4x`yz, w = 2z` – xy. Evaluate a(u, v, w) / a(x, y,z) at the point (1 -1, 0). **(07 Marks)**

c. A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box, if the total surface area is minimum. **(07 Marks)**

Module-3

5 a. Evaluate by changing the order of integration

$$\int fx^2 \bullet dy \bullet dx \text{ , } a > 0$$ **(06 Marks)**

h. Find the area bounded between the circle $x^2 + y^2 = a^2$ and the line x + y = a. **(07 Marks)**

c. Prove that I3(m, F1 • i1 / in + n **(07 Marks)**

OR

6 a. Evaluate $\int_{-c}^{c}\int_{-h}^{h}\int^{a} (x^{-} + y^{-} + z^{-})\, dz.dy.dx$ **(06 Marks)**

b. Find the area bounded by the ellipse $\frac{x}{a^{-}} + \frac{y^2}{b^{-}} = 1$ by double integration. **(07 Marks)**

c. Show that $\int_{o} \frac{dO}{\sqrt{\sin 0}} \times J,/\sin 0.d0 =$ **O** **(07 Marks)**

Module-4

7 a. Solve $(1 + e')dx + e'' \; 1 - \frac{x}{y}\, dy = 0$ **(06 Marks)**

b. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes. Find the temperature of the body after 24 minutes. **(07 Marks)**

c. Solve $yp^2 + (x - y)\, p - x = 0$. **(07 Marks)**

OR

8 a. Solve $\frac{dy}{dx} + y \cdot \tan x = y \cdot \sec x$ **(06 Marks)**

h. Find the orthogonal trajectory of the family of the curves rn•cosnO = a^n, where a is a parameter. **(07 Marks)**

c. Solve the equation (px y) • (py + x) = 2p by reducing into Clairaut's form taking the substitution $X = x^{-}, Y = y^2$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ / & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{pmatrix}$$

by applying elementary Row transformations. **(06 Marks)**

b. Solve the following syste n of equations by Gauss-Jordan method:
$x + y + z = 9,\ 2x + y - z = 0,\ 2x + 5y + 7z = 52$ **(07 Nlarks)**

c. Using Rayleigh's power method find the largest eigen value and corresponding eigen vecto of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ with $X'^{\circ} =$ **(I, 0,** $0)^{1}$ as the initial eigen vector carry out 5 iterations. **(07 Marks)**

OR

10 a. For what values of and t the system of equations.
x + y + z = 6, x+ 2y + 3z = 10, x + 2y + X.z = j_t may have
i) Unique solution ii) Infinite number of solutions iii) No solution. **(06 Marks)**

b. Reduce the matrix $A = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$ into diagonal lbrm. **(07 (harks)**

c. Solve the following system of equations by Gauss-Seidel method
$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25$. Carry out 3 iterations. **(07 Marks)**