CBCS SCHEME

USN 18MAT21

Second Semester B.E. Degree Examination, June/July 2019

Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. If $F = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find div F and curl f. (06 Marks)

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2). (07 Marks)

c. Find the value of a, b, c such that $f = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ is irrotational, also find the scalar potential ϕ such that $F = \nabla\phi$. (07 Marks)

OR

2 a. Find the total work done in moving a particle in the force field $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2+1$, $y = 2t^2$, $z = t^3$ from t = 1 to t = 2. (06 Marks)

b. Using Green's theorem, evaluate $\int_c (xy + y^2)dx + x^2dy$, where C is bounded by y = x and $y = x^2$. (07 Marks)

c. Using Divergence theorem, evaluate $\int F\,ds$, where $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. (07 Marks)

Module-2

3 a. Solve $(D^2 - 3D + 2)y = 2x^2 + \sin 2x$. (06 Marks)

b. Solve $(D^2 +1)y = \sec x$ by the method of variation of parameter. (07 Marks)

c. Solve $x^2y'' - 4xy' + 6y = \cos(2 \log x)$ (07 Marks)

OR

4 a. Solve $(D^2 - 4D + 4)y = e^{2x} + \sin x$. (06 Marks)

b. Solve $(x+1)^2y'' + (x+1)y' + y = 2\sin[\log_e(x+1)]$ (07 Marks)

c. The current i and the charge q in a series containing an inductance L, capacitance C, emf E, satisfy the differential equation $L\frac{d^2q}{dt^2} + \frac{q}{C} = E$, Express q and i interms of 't' given that L, C, E are constants and the value of i and q are both zero initially. (07 Marks)

Module-3

5 a. Form the partial differential equation by elimination of arbitrary function from $\phi(x + y + z, x^2 + y^2 + z^2) = 0$ (06 Marks)

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$ (07 Marks)

c. Derive one dimensional heat equation in the standard form as $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ (07 Marks)

OR

6 a. Solve $\frac{a^{\cdot}}{ax}$ + z = 0 such that z = ey where x = 0 and $\frac{oz}{ax}$ =1 when x = 0. **(06 Marks)**

b. Solve (mz — ny) $\frac{---}{ax}$ + (nx — e) $\frac{az}{aV}$ = ey mx **(07 Marks)**

c. Find all possible solutions of one dimensional wave equation $\frac{a'u}{ate} = c'\frac{a'u}{ax'}$ using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Discuss the nature of the series $\sum_{n=1}^{\infty}\frac{(n+1)^n}{n^{n+1}}x^n$. **(06 Marks)**

b. With usual notation prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ **(07 Marks)**

c. if $x^3 + 2x^2 - x + 1$ = aP3 + bP2 el) ! + dPo , find a, b, c and d using Legendre's polynomial. **(07 Marks)**

OR

8 a. Discuss the nature of the series

$\frac{x}{1.2} \quad \frac{x^2}{3.4} \quad \frac{x}{3.4}$ **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation in terms of $P_n(x)$
$(1-x^2)y'' - 2xy' + n(n+1)y = 0$ **(07 Marks)**

c. Express $x^4 - 3x^2 + x$ interms of Legendre's polynomial. **(07 Marks)**

Module-5

9 a. Find the real root of the equation xsinx + cosx = 0 near x = π using Newton-Raphson method. Carry out 3 iterations. **(06 Marks)**

b. From the following data, find the number of students who have obtained (i) less than 45 marks (ii)between 40 and 45 marks.

Marks	30 — 40	40 — 50	50 — 60	60 — 70	70 - 80
No. of Students	31	42	51	35	31

(07 Marks)

c. Evaluate $\int \frac{1}{1+x^2}$ dx using Simpson's $\frac{3}{8}$th rule by taking 7 ordinates. **(07 Marks)**

OR

10 a. Find the real root of the equation x $\log_{10}$ x = 1.2 which lies between 2 and 3 using Regula-Falsi method. **(06 Marks)**

b. Using Lagrange's int iven data:

x	0	1	2	5
y	2	3	12	147

(07 Marks)

c. Evaluate $\int_4^{5.2}\log_e x\,dx$ using Weddle's rule by taking six equal parts. **(07 Marks)**