



USN

15MAT11

First Semester B.E. Degree Examination, June/July 2019

Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1. a. Find the n^{th} derivative of $\frac{7x+6}{2x^2+7x+6}$ (05 Marks)
 b. Find the angle between the radius vector and the tangent for the curve $r^m = a' (\cos m\theta + \sin m\theta)$. (05 Marks)
 c. Show that the radius of curvature at any point O on the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ is $4a \cos(\theta/2)$ (0 + sing), (06 Marks)

OR

2. a. If $x = \sin t$ and $y = \cos mt$, prove that $(1 - x^2)y_{xx} - 2y_x$ (05 Marks)
 b. Find the pedal equation of the curve $r = a \sec 2\theta$. (05 Marks)
 c. Prove with usual notation $\tan 4\theta = \frac{r \theta}{dr}$. (06 Marks)

Module-2

- a. Expand e^{bx} using Maclaurin's series upto third degree term. (05 Marks)
 b. Evaluate $\lim_{x \rightarrow 0} \left[\frac{1}{x} - \frac{1}{\sin^2 x} \right]$ (05 Marks)
 c. If $u = e^{(a+b)x}$ (ax - by), prove that $b \frac{\partial u}{\partial x} + a \frac{\partial u}{\partial y} = 2abu$ (06 Marks)

OR

4. a. Expand $\sin x$ in ascending power of $t/2$ upto the term containing x^4 . (05 Marks)
 b. If $u = \tan^{-1} \frac{x-y}{x+y}$, show that $x u_x + y u_y = \sin 2u$. (05 Marks)
 c. If $u = y^2$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ (06 Marks)

Module-3

5. a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$. (05 Marks)
 b. Show that $\mathbf{F} = (y+z)\mathbf{i} + (x+z)\mathbf{j} + (x+y)\mathbf{k}$ is irrotational. Also find a scalar function ϕ such that $\nabla \phi = \mathbf{F}$. (05 Marks)
 c. Prove that $\nabla \cdot (\nabla \phi) = (\nabla \phi \cdot \nabla) + \nabla^2 \phi$. (06 Marks)

OR

6. a. Prove that $\text{Curl}(\nabla \phi) = 0$. (05 Marks)
 b. A particle moves along the curve C : $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where t denotes the time. Find the component of acceleration at $t = 2$ along the tangent. (05 Marks)

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- c. Show that $\mathbf{F} = (2xy^2 + yz)\mathbf{i} + (2x^2y + xz + 2yz^2)\mathbf{j} + (2y^2z + xy)\mathbf{k}$ is a conservative force field. Find its scalar potential. (06 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \sin^n x dx$ (05 Marks)
 b. Solve $(y^2 ex''' + 4x^3)dx + (2xye'''' - 3y^2)dy = 0$. (05 Marks)
 c. Find the orthogonal trajectories of $r = a(1+\sin\theta)$. (06 Marks)

OR

- 8 a. Evaluate $\int_1^2 x \cdot \sin 2x - x^2 dx$ (05 Marks)
 b. Solve $(y^3 - 3x^2 y)dx - (x^3 - 3xy^2)dy = 0$. (05 Marks)
 c. A bottle of mineral water at a room temperature of 72°F is kept in a refrigerator where the temperature is 44°F. After half an hour, water cooled to 61°F
 i) What is the temperature of the mineral water in another half an hour?
 ii) How long will it take to cool to 50°F? (06 Marks)

Module-5

- 9 a. Find the rank of the matrix
 $A = \begin{vmatrix} -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{vmatrix}$ (05 Marks)

- b. Find the largest eigen value and corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 2 & 1 \\ 2 & -1 & 3 \end{pmatrix} \text{ by power method taking } X^{(n)} = [1, 1]$$
 (05 Marks)

- c. Reduce the matrix $A = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$ to the diagonal form. (06 Marks)

- 10 a. Use Gauss elimination method to solve
 $2x+y+4z=12$
 $4x+11y-z=33$
 $8x-3y+2z=20$ (05 Marks)

- b. Find the inverse transformation of the following linear transformation.

$$\begin{aligned} y_1 &= x_1 + 2x_2 + 5x_3 \\ y_2 &= 2x_1 + 4x_2 + 11x_3 \\ y_3 &= -x_1 + 2x_2 \end{aligned}$$
 (05 Marks)

- c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3$ to the Canonical form. (06 Marks)