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14MAT11

First Semester B.E. Degree Examination, Dec.2014/Jan.2015
Engineering Mathematics - I

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting ONE full question from each part,

PART - 1

 1 a. If $Y = \cos(m \log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$. (07 Marks)

 b. Find the angle of intersection between the curves $r = a \log \theta$ and $r = \frac{a}{\log \theta}$. (06 Marks)

c. Derive an expression to find radius of curvature in Cartesian form. (07 Marks)

 2 a. If $\sin^{-1} y = 2 \log(x+1)$ prove that $(x^2 + 1)y_{n+2} \pm (2n+1)(x+1)y_{n+1} + (n^2 + 4)y_n = 0$. (07 Marks)

 b. Find the pedal equation $r^3 = \sec hne$. (06 Marks)

 c. $\frac{3}{2} - \frac{-3}{8112}$ (07 Marks)

PART - 2

 3 a. Find the first four non zero terms in the expansion of $f(x) = e^{\frac{3}{x-1}}$. (07 Marks)

 b. If $\cos u = \frac{x+y}{x^2+y^2}$ show that $x \frac{au}{ax} + y \frac{au}{ay} = \frac{\cot u}{2}$. (06 Marks)

 c. Find $\frac{a(u, v, w)}{a(x, y, z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$ and $w = x+y+z$. Hence interpret the result. (07 Marks)

 4 a. If $w = f(x, y)$, $x = r \cos \theta$, $y = r \sin \theta$ show that $\frac{r^2}{ax^2} + \frac{1}{ay^2} - \frac{(aw)^2}{ar^2} = \frac{1}{r^2} \frac{ra-wy}{a\theta}$ (07 Marks)

 b. Evaluate $\int_0^{\pi} (\sin x) dx$. (06 Marks)

 c. Examine the function $f(x, y) = 1 + \sin(x^2 + y^2)$ for extremum. (07 Marks)

PART - 3

 5 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the components of velocity and acceleration at $t = 1$ in the direction $i - 2j + 2k$. (07 Marks)

 b. Using differentiation under integral sign, evaluate $\int_0^1 e^{-ax} \sin x dx$. (07 Marks)

 c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$ (06 Marks)

- 6 a- If $\mathbf{v} = w \times \mathbf{r}$, prove that $\text{curl } \mathbf{v} = 2\mathbf{w}$ where w is a constant vector. (07 Marks)
- b. Show that $\text{div}(\text{curl } \mathbf{A}) = 0$. (06 Marks)
- c. If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{r} = r$. Find $\text{grad div} \left(\frac{\mathbf{r}}{r^3} \right)$. (07 Marks)

PART 4

Obtain the reduction formula for $\int \cos^2 x \, dx$. (07 Marks)

b Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. (06 Marks)

c. Show that the orthogonal trajectories of the family of cardioids $r = a \cos^2 \left(\frac{\theta}{2} \right)$ is another family of cardioids $r = b \sin^2 \left(\frac{\theta}{2} \right)$. (07 Marks)

8 a. Evaluate $\int_0^{\pi} x \sin^2 x \cos x \, dx$. (07 Marks)

b. Solve $\frac{dy}{dx} \tan x = y^2 \sec x$. (06 Marks)

c. If the temperature of the air is 30°C and it cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40° . (07 Marks)

PART 5

9 a. Solve $\begin{cases} x + 2z = 12 \\ x + 2y + 3z = 11 \\ 2x - 2y + z = 2 \end{cases}$ by Gauss elimination method. (06 Marks)

b. Diagonalize the matrix, $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$. (07 Marks)

c. Determine the largest eigen value and the corresponding eigen vector of

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

Starting with $[0, 0, 1]^T$ as the initial eigenvector. Perform 5 iterations. (07 Marks)

10 a. Show that the transformation $y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = 2x_1 + 4x_2 + 11x_3$, $y_3 = -x_1 + 2x_2$ is regular and find the inverse transformation. (06 Marks)

b. Solve by LU decomposition method $2x + y + 4z = 12$, $8x - 3y + 2z = 20$, $4x + 11y + z = 33$. (07 Marks)

c. Reduce the quadratic form $2x^2 + 2y^2 - 2xy - 2yz - 2zx$ into canonical form. Hence indicate its nature, rank, index and signature. (07 Marks)