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15MAT21

Second Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics - II

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full*** *questions, choosing one full question from each module.*

Module-1

1 a. Solve $(D-2)^2 y - 8(e^{2x} + x^2)$, by inverse differential operator method. **(06 Marks)**

b. Solve $(D^2 - 4D + 3)\, y = e^x \cos 2x$, by inverse differential operator method. **(05 Marks)**

c. Solve by the method of variation of parameters $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$. **(05 Marks)**

OR

2 a. Solve $(D^2 - 1)y = x \sin 3x$ by inverse differential operator method. **(06 Marks)**

b. Solve $(D^3 - 6D^2 + 11D - 6)y = e^{2x}$ by inverse differential operator method. **(05 Marks)**

c. Solve $(D^2 + 2D + 4)\, y = 2x^2 + 3\, e^{-x}$ by the method of undetermined coefficient. **(05 Marks)**

Module-2

3 a. Solve $x^3 y''' + 3x^2 y'' + xy' + 8y = 65 \cos(\log x)$. **(06 Marks)**

b. Solve $xy\, p^2 + p(3x^2 - 2y^2) - 6xy = 0$. **(05 Marks)**

c. Solve the equation $y^2(y - xp) = x^4 p^2$ by reducing into Clairaut's form, taking the substitution $x = \frac{1}{X}$ and $y = \frac{1}{Y}$. **(05 Marks)**

OR

4 a. Solve $(2x + 3)^2 y'' - (2x + 3)\, y' - 12y = 6x$. **(06 Marks)**

b. Solve $p^2 + 4x^5 p - 12x^4 y = 0$. **(05 Marks)**

c. Solve $p^3 - 4xy\, p + 8y^2 = 0$. **(05 Marks)**

Module-3

5 a. Obtain the partial differential equation by eliminating the arbitrary function.
$z = f(x + at) + g(x - at)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$ and $z = 0$, when y is an odd multiple of $\frac{\pi}{2}$. **(05 Marks)**

c. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. **(05 Marks)**

OR

6 a. Obtain the partial differential equation by eliminating the arbitrary function
$\phi(x + my + nz - 4)(x^2 + y^2 + z^2)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that, when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$ **(05 Marks)**

c. Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. (05 Marks)

Module-4

7 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x + y + z)\, dy\, dx\, dz$. (06 Marks)

b. Evaluate $\int_{0}^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xy\, dy\, dx$ by changing the order of integration. (05 Marks)

c. Evaluate $\int_0^4 x^{3/2}(4-x)^{5/2}\, dx$ by using Beta and Gamma function. (05 Marks)

OR

8 a. Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ by changing to polar co-ordinates. Hence show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. (06 Marks)

b. Find by double integration, the area lying inside the circle $r = a \sin\theta$ and outside the cardioid $r = a(1 - \cos\theta)$. (05 Marks)

c. Obtain the relation between beta and gamma function in the form $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (05 Marks)

Module-5

9 a. Find i) $L\{e^{3t}(2\cos 5t - 3\sin 5t)\}$ ii) $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. (06 Marks)

b. If a periodic function of period 2a is defined by

$$f(t) = \begin{cases} t & \text{if } 0 \le t \le a \\ 2a - t & \text{if } a \le t \le 2a \end{cases}$$

then show that $L\{f(t)\} = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$. (05 Marks)

c. Solve the equation by Laplace transform method. $y''' + 2y'' - y' - 2y = 0$. Given $y(0) = y'(0) = 0$, $y''(0) = 6$. (05 Marks)

OR

10 a. Find $L^{-1}\left\{\frac{s+3}{s^2 - 4s + 13}\right\}$. (06 Marks)

b. Find $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$ by using Convolution theorem. (05 Marks)

c. Express $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ \sin 2t, & \pi < t < 2\pi \\ \sin 3t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transforms. (05 Marks)