

USN

18MAT11

0.046

First Semester B.E. Degree Examination, Deg.2018/Jan.2019

Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1. a. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ are intersect orthogonally. (06 Marks)
- b. Find the radius of curvature of the curve $y = a \log \sec e^x$ at any point (x, y) . (06 Marks)
- c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. (08 Marks)

OR

2. a. With usual notation, prove that $\text{tart}(1) r \frac{d\theta}{dr}$. (06 Marks)
- b. Find the pedal equation of the curve $r = ae^{c\cot\theta}$. (06 Marks)
- c. Find the radius of curvature for the curve $r = a(1 + \cos\theta)$. (08 Marks)

Module-2

- 3 a. Using Maclaurin's expansion. Prove that $\sin 2x = 2x - \frac{(2x)^3}{6} + \frac{(2x)^5}{120} - \dots$ (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x + d^x - 4}{x^4}$ (07 Marks)
- c. Find the dimensions of the rectangular box open at the top of maximum capacity whose surface is 432 sq.cm. (07 Marks)

OR

4. a. If $u = f(yz, z - x, x - y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(06 Marks)**
- b. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. Find Jacobian $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. **(07 Marks)**
- c. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$. **(07 Marks)**

Module-3

- 5 a. Evaluate $\int_0^1 \int_0^1 e^{-(x+y)} dx dy$, by changing into polar coordinates. (06 Marks)
- b. Find the volume of the tetrahedron bounded by the planes :
 $x = 0, y = 0, z = 0, x + y + z = 1$. (07 Marks)
- c. Prove that $f_3(m, n) = \frac{F(m)F(n)}{F(m+n)}$ (07 Marks)

OR

- 6 a. Evaluate $\int_0^1 \int_x^1 xy dy dx$ by change of order of integration. (06 Marks)
- b. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (x+y+z) dy dx dz$. (07 Marks)
- c. Prove that $\int_0^{\pi} \sin \theta \cdot d\theta \int_0^{\pi} \frac{1}{V \sin \theta} d\theta = \pi$. (07 Marks)

Module-4

- 7 a. A body, in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes. (06 Mark)
- Solve $\frac{dy}{dx} = \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x}$ (07 Marks)
- c. Solve $xyp^2 - (x^2 y^2)p + xy = 0$. (07 Marks)

OR

- 8 a. Solve $\frac{dy}{dx} + y \tan x = y \sec x$. (06 Marks)
- b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. (07 Marks)
- c. Find the general solution of the equation $(px + y)(py + x) = 0$ by reducing into Clairaut's form, taking the substitution $X = x^2, Y = y^2$. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

(07 Marks)

- b. Solve the system of equations

$$12x + y + z = 31$$

$$2x + 8y + z = 24$$

$$3x + 4y + 10z = 58$$

By Gauss —Siedal method.

(07 Marks)

- c. Diagonalize the matrix :

$$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$

(06 Marks)

OR

- 10 a. For what values of X, and M the system of equations :

$$x + 2y + 3z = 6$$

$$x + 3y + 5z = 9$$

$$2x + 5y + z = M$$

has i) no solution ii) a unique solution iii) infinite number of solution.

(07 Marks)

- b. Find the largest eigen value and the corresponding eigen vector of :

$$A = \begin{bmatrix} 6 & -2 \\ -2 & 3 \\ 2 & -1 & 3 \end{bmatrix}$$

by Rayleigh's power method, use $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as the initial eigen vector (carry out 6 iterations).
(07 Marks)

Solve the system of equations :

$$x + y + z = 9$$

$$2x + y - z = 0$$

$$2x + 5y + 7z = 52$$

By Gauss elimination method.

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(06 Marks)