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B.Sc. (Computer Science) (2013 & Onwards) (Sem.–3)

SEQUENCE SERIES AND CALCULUS

Subject Code : BCS-302

M.Code : 71774

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt ANY FOUR questions.**

SECTION-A

1. Write briefly :

a) If a sequence is divergent to ∞, then it is bounded below but not bounded above.

b) Prove that the sequence $\left\langle \frac{1}{n^3} \right\rangle$ is convergent.

c) Show that the series :

$$1 - \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \_\_\_\_\_\_ \text{ is Convergent.}$$

d) State Raabe's test.

e) Prove that the series $\Sigma\, u_n$ is divergent where $u_n = \frac{n}{n+1}$.

f) State the first mean value theorem of integral calculus.

g) State comparison test in limit form for convergence of improper integral $\int_a^b f(x)\,dx$. at a.

h) Show that $\int_0^\infty x^6 e^{-2x}\,dx = \frac{45}{8}$.

i) Express $\int_0^1 x^2 (1-x^3)^{3/2}\,dx$ as a beta function.

j) Compute $\int_{-1}^{1} f\,dx$ where $f(x) = |x|$.

SECTION-B

2. a) Every cauchy sequence of real numbers is convergent.

   b) if $\lim_{n\to\infty} \frac{a_n + 1}{a_n} = l$, where $|l| < 1$, then $\lim_{n\to\infty} a_n = 0$

3. a) Test the convergence of the series $\sum \frac{n^2 - 1}{n^2 + 1} x^n$.

   b) if Σu_n is convergent, show that $\sum \frac{u_n}{1 - u_n} (u_n > 0, u_n \neq 1)$ is also convergent.

4. a) Show that the series :

   $$\frac{1}{\log 2} - \frac{1}{\log 3} - \frac{1}{\log 4} - \frac{1}{\log 5} - \_\_\_\_\_\_$$ is conditionally convergent.

   b) State and prove cauchy's general principle of convergence.

5. a) Prove that if a function is monotonic on [a, b], then show then it is Riemann integrable on [a, b].

   b) If $0 < x < 1$, then show that $\frac{x}{1-x} > \log (1-x)^{-1} > x$.

6. a) Check for convergence the improper integral $\int_0^1 x^{m-1} (1-x)^{n-1}\, dx$ where m, n are real numbers.

   b) State and prove cauchy's test for convergence of $\int_a^b f(x)\, dx$ at a.

7. a) Show that :

   $$\beta (m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx; m > 0, n > 0.$$

   b) Show that $\Gamma (1/2) = \sqrt{\Pi}$.

NOTE : Disclosure of identity by writing mobile number or making passing request on any page of Answer sheet will lead to UMC case against the Student.