Roll No. Total No. of Pages : 02

Total No. of Questions : 07

B.Sc. (Computer Science) (2013 & Onwards) (Sem.–6)

REAL ANALYSIS

Subject Code : BCS-601

M.Code : 72781

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.

SECTION–A

1. Write briefly :

a) State Weierstrass M-test for uniform convergence of sequence of functions.

b) Prove that $f(z) = \overline{z}$ is not analytic anywhere.

c) Show that $f(z) = 1/z$ is not uniformly continuous in the region $|z| < 1$.

d) Show that cross ratio is invariant under bilinear transformation.

e) Determine the angle of rotation at $z = (1 + i)/2$ under the mapping $w = z^2$.

f) Find the radius of convergence of the series $\sum_{n=1}^{\infty} n!\, x^n$.

g) Determine values of a, b such that $z = ax^3 + by^3$ is harmonic function.

h) Prove that $\sum a_n n^{-x}$ is uniformly convergent on $[0,1\}$ if $\sum a_n$ converges uniformly in $[0, 1]$.

i) Discuss the convergence and uniform convergence of sequence $\{e^{-nx}\}$.

j) Determine the linear functional transformation that maps $z_1 = 0$, $z_2 = 0$, $z_3 = 1$ onto $w_1 = -1$, $w_2 = -1$, $w_3 = 1$, respectively.

SECTION-B

2. State and prove Cauchy's General Principle of uniform convergence.

3. Show that the sequence $\{f_n\}$, where $f_n(x)=\dfrac{x}{1+nx^2}$ converges uniformly on R.

4. Examine the convergence of $\int_0^1 x^{n-1}\log x\,dx$

5. a) Prove that necessary condition for $f(z) = u + iv$, $z = x + iy$, to be analytic in a domain D are that $u_x = v_y$ and $u_y = -v_x$.

   b) If $u = e^x (x\cos y - y\sin y)$, find the analytic function $u + i\,v$.

6. If $f(z)$ is an analytic function of z in a region D of the plane and $f'(z) \neq 0$ inside D, show that the maping $w = f(z)$ is conformal at the points of D.

7. Find Fourier expansion of $f(x)=\begin{cases} x-\pi, & -\pi<x<0 \\ \pi-x, & 0<x<\pi \end{cases}$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.