CBCS SCHEME

USN (56)A 15MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020

# Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

## Module-1

1 a. If $y = 2^x \cos^3 x$, find $y_n$. (05 Marks)

b. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)

c. Prove that the curves $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$ cut orthogonally. (05 Marks)

OR

2 a. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. (05 Marks)

b. Find the pedal equation of $r = 2(1+\cos\theta)$. (06 Marks)

c. If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. (05 Marks)

## Module-2

3 a. Expand $\log\cos x$ in powers of $x - \frac{\pi}{3}$ using Taylor's series. (05 Marks)

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$ (06 Marks)

c. If $\sin u = \frac{x^2 y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. (05 Marks)

OR

4 a. Using Maclaurin's series, expand $\log(1+e^x)$ in ascending powers of x. (05 Marks)

b. If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$ prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$. (06 Marks)

c. If $u = x^2+y^2+z^2$, $v = x+y+z$, $w = xy+yz+zx$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ (05 Marks)

## Module-3

5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$, determine the components of velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. (05 Marks)

b. Find the directional derivatives of $\phi = x^2yz + 4xz^2$ at the point (1, -2, -1) along the direction of $2i - j - 2k$. (06 Marks)

c. Prove that $\operatorname{div}(\operatorname{curl} F) = 0$. (05 Marks)

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**OR**

6 a. If $\vec{F} = (3x^2 - 3yz)\hat{i} + (3y^2 - 3zx)\hat{j} + (3z^2 - 3xy)\hat{k}$, find (i) div F (ii) curl F. **(05 Marks)**

b. If $F = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational, find a, b, c. **(06 Marks)**

c. Prove that curl(4A) = (curl A) + V x A **(05 Marks)**

## Module-4

7 a. Find the reduction formula for 'sin' xdx **(05 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$ **(06 Marks)**

c. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}\,dx$. **(05 Marks)**

**OR**

8 a. Find the orthogonal trajectory of the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where $\lambda$ is the parameter. **(05 Marks**

b. Solve $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$. **(06 Marks)**

c. A body in air at 25°C cools from 100°C to 75° in one minute. Find the temperature of the body at the end of three minutes. **(05 Marks)**

## Module-5

9 a. Find the Rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ **(05 Marks)**

b. Apply Gauss-elimination method, to solve the system of equations $x+y+z=9$, $x-2y+3z=8$, $2x+y-z=3$. **(06 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to diagonal form. **(05 Marks)**

**OR**

10 a. Find the largest Eigen value and the corresponding Eigen vector of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and $X = (1\ 0\ 0)'$ as initial vectors. **(05 Marks)**

b. Solve the system of equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$. Carry out the 4th iterations, using Gauss-Seidal method. **(06 Marks)**

c. Reduce the quadratic form of $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz$ into canonical form. **(05 Marks)**