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18MAT21

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Second Semester B.E. Degree Examination, Dec.2019/Jan.2020
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the directional derivative of(= 4xz ^{3} — 3x^{2}y^{2}z at (2, -1, 2) along 2i 3j + 6k . **Marks)**

b. If f = V(x^3 y +7.^{3} —3xyz) find di v f and curl f **(07 Marks)**

c. Find the constants a and b such that F = (axy + z ^{3})i + (3x' — z)j+ (bx y)fc is irrotational. Also find a scalar potential 4:1 if F = V. **(07 Marks)**

OR

2 a- If = xyi + yz j + zxk evaluate F.dr where C is the curve represented by x = t,. y = t $^-$,

1. **(06 Marks)**

b. Using Stoke's theorem Evaluate F.dr if F = (x ^{2} +3/^{2})i —2xy j taken round the rectangle **bounded** by x = 0, x a, y 0, y = b_ **(07 Marks)**

c. Using divergence theorem, evaluate S F.11 ds if F ^{2} — yz)i y zx)j + (z^2 — xy)lc taken around 0 < x 1, 0 < y < 1, 0 < z < 1. **(07 Marks)**

Module-2

3 a. Solve (4D^4 -8D^3 -7D^3 + I1**D** + 6)y = () **(06 Marks)**

b. Solve (D^2 + 4D +3)y = e' **(07 Marks)**

c. Using the method of variation of parameter solve y" + 4y = tan2x. **(07 Marks)**

OR

4 a. Solve (D^3 — 1)y = 3 cos2x **(06 Marks)**

b. Solve x^{2}y" Sxyr + 8y= 2 lOgx **(07 Marks)**

C. The differential equation of a simple pendulum is $\frac{d'x}{dt`}$ + W_0^2x = F_0Sinnt , where W_0 and F_0 are constants. Also initially x = 0, $\frac{dx}{dt}$ = 0 solve it. **(07 Marks)**

Module-3

5 a. Find the PDE by eliminating the function from z = y $^-$ + 2f ($\frac{}{x}$+ log y **(06 Marks)**

b. Solve $\frac{a^2z}{Nay}$ = sin x sin y given ^{Gz} = —2 sin y , when x = 0 and z = 0, when y is odd multiple of $\frac{}{2}$ **(07 Marks)**

c. Derive one-dimensional wave equation in usual notations. **(07 Marks)**

OR

6 a. Solve T a2z x2 = a z given that when x = 0 az/ax = a sin y and z = **0.** **(06 Marks)**

b. Solve x(y – z) p + y (z – x) q = z (x– y). **(07 Marks)**

c. Find all possible solution of $U_t = C^- U$,,, one dimensional heat equation by variable separable method. **(07 Marks)**

Module-4

7 a. Test for convergence for

2^1 31 4^1

2^- 3 $^-$ 4^2 **(06 Marks)**

Find the series solution of Legendre differential equation

$(1 – x^2)$y" - 2xy' + n(n + 1) = 0 leading to P„(x). **(07 Marks)**

c. Prove the orthogonality property of Bessel's function as

$ x j„ (ax) j_n ((3x)dx = 0 a # 1 **(07 Marks)**

0

OR

8 a. Test for convergence for

x-1 1 \ 3/ 2

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Ain , **(06 Marks)**

b. Find the series solution of Bessel differential equation x^2y" + xy' + $(n^2 – x^2)$ **y = 0** Leading to (x) **(07 Marks)**

c. Express the polynomial $x^3 + 2x^2 – 4x + 5$ interms of Legendre polynomials. **(07 Marks)**

Module-5

9 a. Using Newton's forward **(06 Marks)**

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304

b. Find the real root of the equation xlog $_w$ x =1.2 by Regula falsi method between 2 and 3 (Three iterations). **(07 Marks)**

c. Evaluate log x dx by Weddle's rule considering six intervals. (07 **Marks)**

5.2
4

OR

10 a Find t19) from the data by Newton's divided difference formula:

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

(06 Marks)

b. Using Newton – Raphson method, find the real root of the equation x sin x + cosx = 0 near x **(07 Marks)**

c. By using Simpson's rule, evaluate $\int_0^6 \frac{}{1 + x^2}$ by considering seven ordinates. **(07 Marks)**