

# CBCS SCHEME

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17MAT11

## First Semester B.F. Degree Examination, Dec.2019/Jan.2020 Engineering Mathematics — I

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-I

- 1 a. Find the  $n^{\text{th}}$  derivative of  $\sin 2x \cos x$ . (06 Marks)
- b. Prove that the following curves cuts orthogonally  $r = a(1 + \sin \theta)$  and  $r = a(1 - \sin \theta)$ . (07 Marks)
- c. Find the radius of the curvature of the curve  $r = a \sin n\theta$  at the pole. (07 Marks)

### OR

- 2 a. If,  $\tan y = x$ , prove that  $(1 + x^2) \frac{d^2 y}{dx^2} + 2(n+1)xy \frac{dy}{dx} + n(n+1)y = 0$ . (06 Marks)
- b. With usual notations, prove that  $\tan(I) = \frac{WO}{dr}$ . (07 Marks)
- c. Find the radius of curvature for the curve  $n^2 y = a(x^2 + y^2)$  at  $(-2a, 2a)$ . (07 Marks)

### Module-2

- 3 a. Using Maclaurin's series prove that  $\ln(1 + \sin 2x) = x - \frac{x^3}{6} + \frac{x^5}{24} - \dots$  (06 Marks)
- b. If  $U = \cot^{-1} \left( \frac{x+y}{x-y} \right)$ , prove that  $x \frac{dU}{dx} + y \frac{dU}{dy} = -\frac{1}{4} \sin 2U$ . (07 Marks)
- c. Find the Jacobian of  $u = x^2 + y^2 + z^2$ ,  $v = xy + yz + zx$ ,  $w = x + y + z$ . (07 Marks)

### OR

- 4 a. Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ . (06 Marks)
- b. Find the Taylor's series of  $\log(\cos x)$  about the point  $x = \frac{\pi}{2}$  upto the third degree. (07 Marks)
- c. If  $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ . (07 Marks)

### Module-3

- 5 a. If  $x = t^2 + 1$ ,  $y = 4t - 3$ ,  $z = 2t^2 - 6t$  represents the parametric equation of a curve then, find velocity and acceleration at  $t = 1$ . (06 Marks)
- b. Find the constants  $a$  and  $b$  such that  $F = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$  is irrotational. Also find a scalar function  $V$  such that  $F = \nabla V$ . (07 Marks)
- c. Prove that  $\text{div}(\text{curl } A) = 0$ . (07 Marks)

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OR

- 6 a. Find the component of velocity and acceleration for the curve  $r = 2t^2 \mathbf{i} + t^3 \mathbf{j} + (3t - 5)\mathbf{k}$  at the points  $t = 1$  in the direction of  $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ . (06 Marks)
- b. If  $t = V(xy^3z^2)$ , find  $\text{div } t$  and  $\text{curl } t$  at the point  $(1, -1, 1)$ . (07 Marks)
- c. Prove that  $\text{curl}(\text{grad } \phi) = \mathbf{0}$ . (07 Marks)

#### Module-4

- 7 a. Prove that  $\int_0^{\infty} \frac{1}{1+x^2} dx = \frac{\pi}{2}$  using reduction formula. (06 Marks)
- b. Solve  $(x^2 + y + x)dx + xydy = 0$ . (07 Marks)
- c. Find the orthogonal trajectory of  $m = a \sin n\theta$ . (07 Marks)

OR

- 8 a. Find the reduction formula for  $\int \cos^n x dx$  and hence evaluate  $\int \cos^4 x dx$ . (06 Marks)
- b. Solve  $ye'' dx + (w'' + 2y)dy = 0$ . (07 Marks)
- c. A body in air at  $25^\circ\text{C}$  cools from  $100^\circ\text{C}$  to  $75^\circ\text{C}$  in 1 minute. Find the temperature of the body at the end of 3 minutes. (07 Marks)

#### Module-5

- 9 a. Find the rank of the matrix  $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$  by reducing to row echelon form. (06 Marks)
- b. Find the largest eigen and the corresponding eigen vector for  $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$  by taking the initial approximation as  $[1, 0.8, -0.8]^T$  by using power method. Carry out four iterations. (07 Marks)
- c. Show that the transformation  $y_1 = 2x_1 - 2x_2 - x_3$ ,  $y_2 = -4x_1 + 5x_2 + 3x_3$ ,  $y_3 = x_1 - x_2 - x_3$  is regular. Find the inverse transformation. (07 Marks)

OR

- 10 a. Solve the equations  $5x + 2y + z = 12$ ,  $x + 4y + 2z = 15$ ,  $x + 2y + 5z = 20$  by using Gauss Seidal method. Carryout three iterations taking the initial approximation to the solution as  $(1, 0, 3)$ . (06 Marks)
- b. Diagonalize the matrix  $A = \begin{bmatrix} 3 & \\ & 4 \end{bmatrix}$ . (07 Marks)
- c. Reduce the quadratic form  $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$  into canonical form by orthogonal transformation. (07 Marks)