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18MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Transform Calculus, Fourier Series and Numerical
Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform of:

(i) $\frac{4t+5}{e^{2t}}$

(ii) $\sin 2t$

(iii) $t \cos at$

(10 Marks)

- b. The square wave function $f(t)$ with period $2a$ defined by $f(t) = \begin{cases} 1 & 0 \leq t < a \\ -1 & a \leq t < 2a \end{cases}$. Show that

$$\frac{-j \tanh \frac{as}{2}}{s}$$

(05 Marks)

- c. Employ Laplace transform to solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} = 0$, $y(0) = y'(0) = 3$.

(05 Marks)

OR

2 a. Find (i) $\int_0^\infty \frac{-3s+4}{s^3} e^{-st} dt$

(ii) $\cot^{-1} \frac{t}{2}$

(iii)

$$\frac{1}{(s+2)(s+3)}$$

(10 Marks)

- b. Find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$ using convolution theorem.

(05 Marks)

c. Express $f(t) = \begin{cases} 2 & \text{if } 0 < t < 1 \\ 2 & \text{if } 1 < t < \frac{11}{2} \\ \cos t & t > \frac{11}{2} \end{cases}$ in terms of unit step function and hence find its Laplace transformation.

(05 Marks)

Module-2

- 3 a. Obtain the Fourier series of $f(x) = \begin{cases} 2 & -2 < x < 0 \\ x & 0 < x < 2 \end{cases}$

(08 Marks)

- b. Find the half range cosine series of $f(x) = (x+1)$ in the interval $0 < x < \pi$

(06 Marks)

- c. Express $f(x) = x$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$.

(06 Marks)

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OR

- 4 a. Compute the first two harmonics of the Fourier Series of $f(x)$ given the following table :

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

(08 Marks)

- b. Find the half range size series of e^x in the interval $0 < x$

(06 Marks)

- c. Obtain the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x}{4}$ valid in the interval $(-t, n)$

(06 Marks)

Module-3

- 5 a. Find the Infinite Fourier transform of e^{-x}

(07 Marks)

- b. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$.

(06 Marks)

- c. Solve $u_{xx} - 3u_{xy} + 2u_y = 3$, given $u_x = u_y = 0$.

(07 Marks)

OR

- 6 a. If $f(x) = \begin{cases} 11 \text{ for } 0 < x < a \\ 0 \text{ for } x > a \end{cases}$, find the infinite transform of $f(x)$ and hence evaluate $\int_0^\infty f(x) dx$.

(07 Marks)

- b. Obtain the Z-transform of $\cosh nx$ and $\sinh nx$.

(06 Marks)

- c. Find the inverse Z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$

(07 Marks)

Module-4

- 7 a. Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's Series method upto 4th degree terms and find the value of $y(1.1)$.

(07 Marks)

- b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} + y = 2x$ at $x = 1.1$ given $y(1) = 3$ (Take $h = 0.1$)

(06 Marks)

- c. Apply Milne's predictor-corrector formulae to compute $y(0.4)$ given $\frac{dy}{dx} = 2exy$, with

(07 Marks)

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

OR

- 8 a. Given $\frac{dy}{dx} = x + \sin y$, $y(0) = 1$. Compute $y(0.4)$ with $h = 0.2$ using Euler's modified method.

(07 Marks)

- b. Apply Runge-Kutta fourth order method, to find $y(0.1)$ with $h = 0.1$ given $\frac{dy}{dx} + y + xy^2 = 0$; $y(0) = 1$.

(06 Marks)

- c. Using Adams-Bashforth method, find $y(4.4)$ given $5x \frac{dy}{dx} = 2$ with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

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Module-5

- 9 a. Solve by Runge Kutta method $\frac{d^2y}{dx^2} = x \frac{dy}{dx} - y$, for $x = 0.2$ correct 4 decimal places, using initial conditions $y(0) = 1$, $y'(0) = 0$, $h = 0.2$. **(07 Marks)**
- b. Derive Euler's equation in the standard $\frac{d}{dy} \frac{d}{dx} \text{ of } = 0$. **(06 Marks)**
- c. Find the extremal of the functional, $\int (y')^2 + 2ye' dx$ **(07 Marks)**

OR

- 10 a. Apply Milne's predictor corrector method to compute $\frac{d^2y}{dx^2}$ and the following table of initial values:

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

(07 Marks)

- b. Find the extremal for the functional, $\int_0^1 2y \sin x \, dx$; $y(0) = 1$, $y(1) = 1$. **(06 Marks)**
- c. Prove that geodesics of a plane surface are straight lines. **(07 Marks)**