CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the Laplace transform of:

(i) $\left(\frac{4t+5}{e^{2t}}\right)^2$ (ii) $\left(\sin 2t\right)^2$ **(iii) t cos at .** **(10 Marks)**

b. The square wave function f(t) with period 2a defined by $f(t) = \begin{cases} 1 & 0 \le t < a \\ -1 & a \le t < 2a \end{cases}$. Show that

$\frac{1}{s}\tanh\left(\frac{as}{2}\right)$ **(05 Marks)**

c. Employ Laplace transform to solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} = 0$, y(0) = y, (o) = 3. **(05 Marks)**

OR

2 a. Find (i) $\left\{\frac{s^2 - 3s + 4}{s^3}\right\}$ (ii) $\cot^{-1}\left(\frac{s}{2}\right)$ (iii) $\frac{}{(s+2)(s+3)}$ **(10 Marks)**

b. Find the inverse Laplace transform of. $\frac{1}{s(s^2+1)}$ using convolution theorem. **(05 Marks)**

c. Express $f(t) = \begin{cases} 2 & \text{if } 0 < t < 1 \\ \frac{t^2}{2} & \text{if } 1 < t < \frac{\pi}{2} \\ \cos t & t > \frac{\pi}{2} \end{cases}$ in terms of unit step function and hence find its Laplace transformation. **(05 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x) = \begin{cases} 2 & -2 < x < 0 \\ x & 0 < x < 2 \end{cases}$ **(08 Marks)**

b. Find the half range cosine series of, $f(x) = (x+1)^2$ in the interval 0 x **(06 Marks)**

c. Express f(x) = x as a Fourier series of period 27 in the interval 0 < x < 2rc . **(06 Marks)**

OR

4 a. Compute the first two harmonics of the Fourier Series of f(x) given the following table :

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

(08 Marks)

b. Find the half range size series of e^x in the interval 0 < x **(06 Marks)**

C. Obtain the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x}{4}$ valid in the interval (–t n) **(06 Marks)**

Module-3

5 a. Find the Infinite Fourier transform of e - **(07 Marks)**

b. Find the Fourier cosine transform of $f(x) = e^{-2} " + 4e^{-3} "$. **(06 Marks)**

c. Solve u,,,₂ –3u,,_, + 2u,, = 3" , given u, = u, = 0 . **(07 Marks)**

OR

6 a. If f(x) = $\begin{cases} 1 \text{ for } |x| \le a \\ 0 \text{ for } |x| > a \end{cases}$ find the infinite transform of f(x) and hence evaluate ______ dx .

(07 Marks,_

b. Obtain the Z-transform of cosh nO and sinh ne . **(06 Marks)**

e, Find the inverse Z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ **(07 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx}$ = e' – y , y(0) = 2 using Taylor's Series method upto 4 th degree terms and find the value of y(1.1). **(07 Marks)**

b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx}$ + y = 2x at x = 1.1 given y(1) = 3 (Take h 0.1) **(06 Marks)**

c. Apply Milne's predictor-corrector formulae to compute y(0.4) given $\frac{dy}{dx}$ = 2ex y , with **(07 Marks)**

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

OR

8 a. Given $\frac{dy}{dx} = x + \sin y$ y(0) = 1. Compute y(0.4) with h = 0.2 using Euler's modified method. **(07 Marks)**

b. Apply Runge-Kutta fourth order method, to find y(0.1) with h = 0.1 given $\frac{dy}{dx} + y + xy^2 = 0$; y(0) = I. **(06 Marks)**

C. Using Adams-Bashforth method, find y(4.4) given 5x $\frac{dy}{dx}$ = 2 with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

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Module-5

9 a. Solve by Runge Kutta method $\frac{d'y}{dx^2} = x\frac{dy}{dx} - y$, for x = 0.2 correct 4 decimal places, using initial conditions y(0) = 1, y'(0) = 0, h = 0.2. **(07 Marks)**

b. Derive Euler's equation in the standard $\frac{}{y} \frac{d}{dx} \frac{of}{cry} = 0$. **(06 Marks)**

c. Find the extramal of the functional, +(y') + 2ye'dx **(07 Marks)**

OR

10 a. Apply Milnes predictor corrector method to compute $\frac{d}{dx} y \qquad \frac{dy}{dx}$ and the following table of initial values:

x	0	0.1	0.2	0.3
y	1	1.1.1.03	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

(07 Marks)

b- Find the extramal for the functional. $-y^{f2}$ - 2y sin x ix ; y(0) — yr =1. **(06 Marks)**

c. Prove that geodesics of a plane surface are straight lines. **(07 Marks)**