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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If [I, ,1n, , n_i l and [1_2 , m_2 n_2] be the direction cosines of two lines subtending an angle **0** between them then prove that cos 0 = 1_jl + m, m_2 +n_i n,. **(06 Marks)**

b. Find the angle between two lines whose direction cosines satisfy the relations 1+ m + n = **0** and 21m + 2n1 — mn =0 **(07 Marks)**

c. Find the co-ordinates of the foot of the perpendicular from A(1, 1,1) to the line joining B(1, 4, 6) and C(5, 4, 4) . **(07 Marks)**

2 a. Find the equation of the plane which bisects the line joining (3, 0, 5) and (1, 2, —1) at right angles. **(06 Marks)**

b. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, —1, —1) are coplanar. Find the equation of the plane containing them. **(07 Marks)**

c. Find the shortest distance and the equations of the line of shortest distance between the lines: $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle a = 4i + 5j + 6k , b = 51+ 6J + 4k and c = 61+ 41+ 5k form an isosceles triangle. **(06 Marks)**

b. Prove that the points with position vectors 4i +53+ j+ 31+ 93+ 41(and —**i+ Sj+4k** are coplanar. **(07 Marks)**

c. A particle moves along the curve x 2t^2, y = t ^ — 4t and z = 3t 5 where t is the time t. Find the components of velocity and acceleration in the direction of the vector i — 3 j + 2k at t 1. **(07 Marks)**

4 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 - = 3$ at (2,-1, 2). **(06 Marks)**

b. Find the directional derivatives of the function (I) = x yz + 4xz ^{2} at (1,-2,-1) along 2i — j— 2k **(07 Marks)**

c. Find div F and curl F at the point (1,-1, 1) where F = NIxy ^{3}z ^{2}). **(07 Marks)**

5 a. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ then prove that,

(i) V(ril)= nrn^{-2} r V.(rn. r + 3)rn **(06 Marks)**

b. Show that F = 2xy ^{2} + yz)i + (2x ^{2}y+ xz + 2yz ^{2})1+ (2 y ^{2}z + xy)fc is irrotational and hence find a scalar function (i) such that F = Vcb . **(07 Marks)**

c. Find the value of the constant 'a' such that A = y(ax ^{2} + z) + x(y)3 + 2xy(z — xy) k is Solenoidal. For this value of 'a' show that curl A is also solenoidal. **(07 Marks)**

6 a. Find the Laplace transform of, (i) sin 5t cos 2t (ii) $(3t + 2)^2$ **(06 Marks)**

b. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$ **(07 Marks)**

c. Find the Laplace transform of $t^2 \sin at$. **(07 Marks)**

7 a. Find the inverse Laplace transform of $\frac{s + 5}{s^2 - 6s + 13}$ **(06 Marks)**

b. Find $L^{-1}\{\log\frac{s + a}{s + b}\}$ **(07 Marks)**

c. Find **(07 Marks)**

8 a. Using convolution theorem find the Laplace transform of $\frac{1}{s(s^2 + a^2)}$. **(10 Marks)**

b. Solve the differential equation, $y'' + 5y' + 6y = 5e^{2t}$ under the condition $y(0) = 2$, $y'(0) = 1$ using Laplace transform. **(10 Marks)**