

CBCS SCHEM

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17MAT41

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020 Engineering Mathematics - IV

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. From Taylor's series method, find $y(0.1)$, considering upto fourth degree term if $y(x)$ satisfying the equation $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. (06 Marks)
- b. Using Runge-Kutta method of fourth order $\frac{dy}{dx} + y = 2x$ at $x = 1.1$ given that $y = 3$ at $x = 1$ initially. (07 Marks)
- c. If $\frac{dy}{dx} = 2ex - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$, find $y(0.4)$ correct upto four decimal places by using Milne's predictor-corrector formula. (07 Marks)

OR

- 2 a. Using modified Euler's method find y at $x = 0.2$ given $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with $y(0) = 1$ taking $h = 0.1$. (06 Marks)
- b. Given $\frac{dy}{dx} + y + zy^2 = 0$ and $y(0) = 1$, $y(0.1) = 0.9008$, $y(0.2) = 0.8066$, $y(0.3) = 0.722$. Evaluate $y(0.4)$ by Adams-Bashforth method. (07 Marks)
- c. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$ taking $h = 0.2$. (07 Marks)

Module-2

- 3 a. Apply Milne's method to compute $y(0.8)$ given that $\frac{d^2y}{dx^2} = 1 - 2y$ and the following table of initial values.

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

- b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (06 Marks)
- c. Obtain the series solution of Bessel's differential equation $x^2 y'' + xy' + (x^2 + n^2)y = 0$ leading to $J_n(x)$. (07 Marks)

OR

- 4 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0$, compute $y(0.2)$ using fourth order Runge-Kutta method. (06 Marks)
- b. Prove $J_{-1/2}(k) = \sqrt{\frac{2}{\pi}} \cos k$. (07 Marks)
- c. Prove the Rodrigues formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. (07 Marks)

Module 3

- 5 a. Derive Cauchy-Riemann equations in Cartesian form. (06 Marks)
- b. Discuss the transformation $w = z^2$. (07 Marks)
- c. By using Cauchy's residue theorem, evaluate $\oint_C \frac{e^{2z}}{(z+1)(z+2)} dz$ if C is the circle $|z| = 3$. (07 Marks)

OR

- 6 a. Prove that $\frac{a}{e^x} + \frac{b}{ay^e}$. If $|f(z)| = 4|f'(z)|$. (06 Marks)
- b. State and prove Cauchy's integral formula. (07 Marks)
- c. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$. (07 Marks)

Module 4

- 7 a. Find the mean and standard of Poisson distribution. (06 Marks)
- b. In an examination 7% of students score less than 35 marks and 89% of the students score less than 60 marks. Find the mean and standard deviation if the marks are normally distributed given $A(1.2263) = 0.39$ and $A(1.4757) = 0.43$. (07 Marks)
- c. The joint probability distribution

$V \backslash$	-2	-1	4	
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine:

- Marginal distribution of X and Y
- Covariance of X and Y
- Correlation of X and Y

(07 Marks)

OR

- 8 a. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K ²	2K ²	7K ² +K

Find K and evaluate $P(x=6) - P(3 < x < 6)$.

(06 Marks)

- b. The probability that a pen manufactured by a factory be defective is $1/10$. If 12 such pens are manufactured, what is the probability that
- Exactly 2 are defective
 - Atleast two are defective
 - None of them are defective.
- (07 Marks)
- c. The length of telephone conversation in a booth has been exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made
- Ends in less than 5 minutes
 - Between 5 and 10 minutes.

(07 Marks)

17MAT41

Module-5

- 9 a. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die cannot be regarded as an unbiased die. (06 Marks)
- b. A group of 10 boys fed on diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weight (lbs):

Diet A:	5	6	8	1	12	4	3	9	6	10
Diet B:	2	3	6	8	10	1	2	8		

 Test whether diets A and B differ significantly $t_{0.05} = 2.12$ at 16df

(07 Marks)

- c. Find the unique fixed probability vector for the regular stochastic matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}$$

(07 Marks)

OR

- 10 a. Define the terms:
- Null hypothesis
 - Type-I and Type-II error
 - Confidence limits
- (06 Marks)
- b. The t.p.m. of a Markov chain is given by $P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \end{pmatrix}$. Find the fixed probabilities vector. (07 Marks)
- c. Two boys B1 and B2 and two girls G1 and G2 are throwing ball from one to another. Each boy throws the ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand each girl throws the ball to each boy with probability 1/2 and never to the other girl. In the long run how often does each receive the ball? (07 Marks)