



USN

15MAT31

Third Semester B.F. Degree Examination, Dec..24*Jan.2020
Engineering Mathematics — III

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-I

- 1 a. Obtain the Fourier expansion of the function $f(x) = x$ over the interval $(-7c, 7c)$. Deduce that $\frac{1}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ (08 Marks)
- b. The following table gives the variations of a periodic current A over a certain period T:

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the first harmonic. (08 Marks)

OR

- 2 a. Obtain the Fourier series for the function $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi/2, & \pi/2 < x < \pi \end{cases}$ (06 Marks)
- b. Represent the function $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi/2, & \pi/2 < x < \pi \end{cases}$ in a half range Fourier sine series. (05 Marks)
- c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following table:

x°	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

(05 Marks)

Module-2

- 3 Find the complex Fourier transform of the function $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$. Hence evaluate $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$. (06 Marks)
- b. If $u(z) = \frac{3z+12}{(z-1)^2}$ show that $u_0 = 0$, $u_1 = 0$, $u_2 = 2$, $u_3 = 11$. (05 Marks)
- c. Obtain the Fourier cosine transform of the function $f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$. (05 Marks)

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OR

- 4 a. Obtain the Z-transform of $\cos n\theta$ and $\sin n\theta$. (06 Marks)
- b. Find the Fourier sine transform of $f(x) = \frac{x}{1+x^2}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$ $m > 0$. (05 Marks)
- c. Solve by using Z-transforms $y_{n+2} + 2y_{n+1} - y_n = n$ with $y_0 = 0 = y_1$. (05 Marks)

Module-3

- 5 a. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at $x = 6$. (06 Marks)

		2	3	4	5
y	10	12	13	16	19

- b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

- c. Use Newton-Raphson method to find a real root of $x \sin x + \cos x = 0$ near $x = \pi$. Carry out the upto four decimal places of accuracy. (05 Marks)

OR

- 6 a. Show that a real root of the equation $\tan x + \tanh x = 0$ lies between 2 and 3. Then apply the Regula Falsi method to find third approximation. (06 Marks)
- b. Compute the coefficient of correlation and the equation of the lines of regression for the data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

- c. Fit a curve of the form $y = ae^{bx}$ for the data:

x	0	2	4
y	8.12	10	31.82

Module-4

- 7 a. From the following table find the number of students who have obtained:
- Less than 45 marks
 - Between 40 and 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
Number of students	31	42	51	35	31

- b. Construct the interpolating polynomial for the data given below using Newton's general interpolation formula for divided differences and hence find y at $x = 3$. (06 Marks)

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

- c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule. Taking seven ordinates. Hence find $\log_e 2$. (05 Marks)

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OR

- 8 a. Use Lagrange's interpolation formula to find $f(4)$ given below. (06 Marks)

				6
$f(x)$		11	14	158

- b. Use Simpson's $3/8$ rule to evaluate $\int_0^4 e^x dx$ (05 Marks)

- c. The area of a circle (A) corresponding to diameter (D) is given by

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

(05 Marks)

Module-5

- 9 a. Evaluate Green's theorem for $\int_C (xy + y^2) dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (06 Marks)
- b. Find the extrema of the functional $f(x, y, z) = x^2 + 3y^2 + 2z^2 + 2xy$. (05 Marks)
- c. Verify Stoke's theorem for $\mathbf{F} = (2x - y)\mathbf{i} + yz^2\mathbf{j} - y^2z\mathbf{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, C is its boundary. (05 Marks)

OR

- 10 a. Derive Euler's equation in the standard form $\frac{d}{dx} \left(\frac{\partial f}{\partial y_1} \right) = \frac{\partial f}{\partial y_1}$. (06 Marks)
- b. If $\mathbf{F} = 2xy\mathbf{i} + 3x^2y\mathbf{j} + xz\mathbf{k}$ and S is the rectangular parallelepiped bounded by $x = 0, y = 0, z = 0, x = 2, y = 1, z = 3$. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$. (05 Marks)
- c. Prove that the shortest distance between two points in a plane is along the straight line joining them or prove that the geodesics on a plane are straight lines. (05 Marks)