

# CBCS SCHEM

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**17MAT41**

## Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020 Engineering Mathematics - IV

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing  
ONE full question from each module.*

### Module-1

- 1 a. From Taylor's series method, find  $y(0.1)$ , considering upto fourth degree term if  $y(x)$  satisfying the equation  $\frac{dy}{dx} = x - y^2$ ,  $y(0) = 1$ . (06 Marks)
- b. Using Runge-Kutta method of fourth order  $\frac{dy}{dx} + y = 2x$  at  $x = 1.1$  given that  $y = 3$  at  $x = 1$  initially. (07 Marks)
- c. If  $\frac{dy}{dx} = 2ex - y$ ,  $y(0) = 2$ ,  $y(0.1) = 2.010$ ,  $y(0.2) = 2.040$  and  $y(0.3) = 2.090$ , find  $y(0.4)$  correct upto four decimal places by using Milne's predictor-corrector formula. (07 Marks)

### OR

- 2 a. Using modified Euler's method find  $y$  at  $x = 0.2$  given  $\frac{dy}{dx} = 3x + \frac{1}{2}y$  with  $y(0) = 1$  taking  $h = 0.1$ . (06 Marks)
- b. Given  $\frac{dy}{dx} + y + zy^2 = 0$  and  $y(0) = 1$ ,  $y(0.1) = 0.9008$ ,  $y(0.2) = 0.8066$ ,  $y(0.3) = 0.722$ . Evaluate  $y(0.4)$  by Adams-Bashforth method. (07 Marks)
- c. Using Runge-Kutta method of fourth order, find  $y(0.2)$  for the equation  $\frac{dy}{dx} = \frac{y - x}{y + x}$ ,  $y(0) = 1$  taking  $h = 0.2$ . (07 Marks)

### Module-2

- 3 a. Apply Milne's method to compute  $y(0.8)$  given that  $\frac{d^2y}{dx^2} = 1 - 2y$  and the following table of initial values.

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

- b. Express  $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$  in terms of Legendre polynomials. (06 Marks)
- c. Obtain the series solution of Bessel's differential equation  $x^2 y'' + xy' + (x^2 + n^2)y = 0$  leading to  $J_n(x)$ . (07 Marks)

OR

- 4 a. Given  $y'' - xy' - y = 0$  with the initial conditions  $y(0) = 1, y'(0) = 0$ , compute  $y(0.2)$  using fourth order Runge-Kutta method. (06 Marks)
- b. Prove  $J_{-1/2}(k) = \sqrt{\frac{2}{\pi k}} \cos x$ . (07 Marks)
- c. Prove the Rodrigues formula  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ . (07 Marks)

### Module 3

- 5 a. Derive Cauchy-Riemann equations in Cartesian form. (06 Marks)
- b. Discuss the transformation  $w = z^2$ . (07 Marks)
- c. By using Cauchy's residue theorem, evaluate  $\oint_C \frac{e^{2z}}{(z+1)(z+2)} dz$  if  $C$  is the circle  $|z| = 3$ . (07 Marks)

OR

- 6 a. Prove that  $\frac{a}{e^x} + \frac{b}{ay^e}$ . If  $|z| = 4$  find  $|f(z)|$ . (06 Marks)
- b. State and prove Cauchy's integral formula. (07 Marks)
- c. Find the bilinear transformation which maps  $z = \infty, i, 0$  into  $w = -1, -i, 1$ . (07 Marks)

### Module 4

- 7 a. Find the mean and standard of Poisson distribution. (06 Marks)
- b. In an examination 7% of students score less than 35 marks and 89% of the students score less than 60 marks. Find the mean and standard deviation if the marks are normally distributed given  $A(1.2263) = 0.39$  and  $A(1.4757) = 0.43$ . (07 Marks)
- c. The joint probability distribution

V \	-2	-1	4	
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine:

- Marginal distribution of X and Y
- Covariance of X and Y
- Correlation of X and Y

(07 Marks)

OR

- 8 a. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K <sup>2</sup>	2K <sup>2</sup>	7K <sup>2</sup> +K

Find K and evaluate  $P(x \leq 6) - P(3 < x < 6)$ .

(06 Marks)

- b. The probability that a pen manufactured by a factory be defective is  $1/10$ . If 12 such pens are manufactured, what is the probability that
- Exactly 2 are defective
  - Atleast two are defective
  - None of them are defective.
- (07 Marks)
- c. The length of telephone conversation in a booth has been exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made
- Ends in less than 5 minutes
  - Between 5 and 10 minutes.

(07 Marks)

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**Module-5**

- 9 a. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die cannot be regarded as an unbiased die. (06 Marks)
- b. A group of 10 boys fed on diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weight (lbs):

Diet A:	5	6	8	1	12	4	3	9	6	10
Diet B:	2	3	6	8	10	1	2	8		

 Test whether diets A and B differ significantly  $t_{0.05} = 2.12$  at 16df

(07 Marks)

- c. Find the unique fixed probability vector for the regular stochastic matrix

$$\begin{pmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}$$

(07 Marks)

**OR**

- 10 a. Define the terms:
- Null hypothesis
  - Type-I and Type-II error
  - Confidence limits
- (06 Marks)
- b. The t.p.m. of a Markov chain is given by  $P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \end{pmatrix}$ . Find the fixed probabilities vector. (07 Marks)
- c. Two boys B1 and B2 and two girls G1 and G2 are throwing ball from one to another. Each boy throws the ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand each girl throws the ball to each boy with probability 1/2 and never to the other girl. In the long run how often does each receive the ball? (07 Marks)