

**CBCS SCHEME**

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**18MAT31**

**Third Semester B.E. Degree Examination, Dec.2019/Jan.2020**  
**Transform Calculus, Fourier Series and Numerical**  
**Techniques**

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

**Module-1**

- 1 a. Find the Laplace transform of:

(i)  $\frac{4t+5}{e^{2t}}$  (ii)  $\sin 2t$

(iii)  $t \cos at$

(10 Marks)

- b. The square wave function  $f(t)$  with period  $2a$  defined by  $f(t) = \begin{cases} 1 & 0 \leq t < a \\ -1 & a \leq t < 2a \end{cases}$ . Show that

$$\frac{-j \tanh \frac{as}{2}}{s}$$

(05 Marks)

- c. Employ Laplace transform to solve  $\frac{d^2y}{dt^2} + \frac{dy}{dt} = 0$ ,  $y(0) = y'(0) = 3$ .

(05 Marks)

**OR**

2 a. Find (i)  $\int_0^\infty \frac{3s+4}{s^3} e^{-st} dt$

(ii)  $\cot^{-1} \frac{1}{2}$

(iii)  $\frac{1}{(s+2)(s+3)}$

(10 Marks)

- b. Find the inverse Laplace transform of  $\frac{1}{s(s^2+1)}$  using convolution theorem.

(05 Marks)

c. Express  $f(t) = \begin{cases} 2 & \text{if } 0 < t < 1 \\ 2 & \text{if } 1 < t < \frac{11}{2} \\ \cos t & t > \frac{11}{2} \end{cases}$  in terms of unit step function and hence find its Laplace transformation.

(05 Marks)

**Module-2**

- 3 a. Obtain the Fourier series of  $f(x) = \begin{cases} 2 & -2 < x < 0 \\ x & 0 < x < 2 \end{cases}$

(08 Marks)

- b. Find the half range cosine series of  $f(x) = (x+1)$  in the interval  $0 < x < \pi$

(06 Marks)

- c. Express  $f(x) = x$  as a Fourier series of period  $2\pi$  in the interval  $0 < x < 2\pi$ .

(06 Marks)

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**OR**

- 4 a. Compute the first two harmonics of the Fourier Series of  $f(x)$  given the following table :

|           |     |     |      |      |      |      |
|-----------|-----|-----|------|------|------|------|
| $x^\circ$ | 0   | 60° | 120° | 180° | 240° | 300° |
| y         | 7.9 | 7.2 | 3.6  | 0.5  | 0.9  | 6.8  |

(08 Marks)

- b. Find the half range size series of  $e^x$  in the interval  $0 < x$

(06 Marks)

- c. Obtain the Fourier series of  $f(x) = \frac{\pi^2}{12} - \frac{x}{4}$  valid in the interval  $(-t, n)$

(06 Marks)

### Module-3

- 5 a. Find the Infinite Fourier transform of  $e^{-x}$

(07 Marks)

- b. Find the Fourier cosine transform of  $f(x) = e^{-2x} + 4e^{-3x}$ .

(06 Marks)

- c. Solve  $u_{xx} - 3u_{xy} + 2u_y = 3$ , given  $u_x = u_y = 0$ .

(07 Marks)

**OR**

- 6 a. If  $f(x) = \begin{cases} 11 & \text{for } 0 < x < a \\ 0 & \text{for } x > a \end{cases}$ , find the infinite transform of  $f(x)$  and hence evaluate  $\int_0^\infty f(x) dx$ .

(07 Marks)

- b. Obtain the Z-transform of  $\cosh nx$  and  $\sinh nx$ .

(06 Marks)

- c. Find the inverse Z-transform of  $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$

(07 Marks)

### Module-4

- 7 a. Solve  $\frac{dy}{dx} = e^x - y$ ,  $y(0) = 2$  using Taylor's Series method upto 4<sup>th</sup> degree terms and find the value of  $y(1.1)$ .

(07 Marks)

- b. Use Runge-Kutta method of fourth order to solve  $\frac{dy}{dx} + y = 2x$  at  $x = 1.1$  given  $y(1) = 3$  (Take  $h = 0.1$ )

(06 Marks)

- c. Apply Milne's predictor-corrector formulae to compute  $y(0.4)$  given  $\frac{dy}{dx} = 2exy$ , with

(07 Marks)

|   |     |       |       |       |
|---|-----|-------|-------|-------|
| x | 0   | 0.1   | 0.2   | 0.3   |
| y | 2.4 | 2.473 | 3.129 | 4.059 |

**OR**

- 8 a. Given  $\frac{dy}{dx} = x + \sin y$ ,  $y(0) = 1$ . Compute  $y(0.4)$  with  $h = 0.2$  using Euler's modified method.

(07 Marks)

- b. Apply Runge-Kutta fourth order method, to find  $y(0.1)$  with  $h = 0.1$  given  $\frac{dy}{dx} + y + xy^2 = 0$ ;  $y(0) = 1$ .

(06 Marks)

- c. Using Adams-Bashforth method, find  $y(4.4)$  given  $5x \frac{dy}{dx} = 2$  with

|   |   |        |        |        |
|---|---|--------|--------|--------|
| x | 4 | 4.1    | 4.2    | 4.3    |
| y | 1 | 1.0049 | 1.0097 | 1.0143 |

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**Module-5**

- 9 a. Solve by Runge Kutta method  $\frac{d^2y}{dx^2} = x \frac{dy}{dx} - y$ , for  $x = 0.2$  correct 4 decimal places, using initial conditions  $y(0) = 1$ ,  $y'(0) = 0$ ,  $h = 0.2$ . **(07 Marks)**
- b. Derive Euler's equation in the standard  $\frac{d}{dy} \frac{d}{dx} \text{ of } = 0$ . **(06 Marks)**
- c. Find the extremal of the functional,  $\int (y')^2 + 2ye' dx$  **(07 Marks)**

**OR**

- 10 a. Apply Milne's predictor corrector method to compute  $\frac{d^2y}{dx^2}$  and the following table of initial values:

|    |   |        |        |        |
|----|---|--------|--------|--------|
| x  | 0 | 0.1    | 0.2    | 0.3    |
| y  | 1 | 1.1103 | 1.2427 | 1.3990 |
| y' | 1 | 1.2103 | 1.4427 | 1.6990 |

**(07 Marks)**

- b. Find the extremal for the functional.  $\int_0^1 2y \sin x \, dx$ ;  $y(0) = 1$ ,  $y(1) = 1$ . **(06 Marks)**
- c. Prove that geodesics of a plane surface are straight lines. **(07 Marks)**