Roll No. | | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 09

B.Sc. (Non Medical) (2018 & Onwards) (Sem.–1)

DIFFERENTIAL CALCULAS

Subject Code : BSNM-105-18

M.Code : 75746

Time : 3 Hrs. Max. Marks : 50

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying ONE mark each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Answer briefly :

a) Define sequence.

b) Define limit inferior with example.

c) Define Left Hand Limit.

d) Define uniform continuity.

e) Define Right hand derivatives.

f) Find $\frac{\partial(f,g)}{\partial(x,y)}$ if $f = x^2 - x\sin y$ and $g = x^2 y^2 + x + y$

g) Show that the function $f(x, y) = |x| + |y|$ is continuous at the origin.

h) State Euler's Theorem on homogeneous function.

i) Prove that a real polynomial function is continuous everywhere.

j) Give an example of a decreasing sequence which diverges to $-\infty$.

SECTION-B

2. State and prove Cauchy's first theorem on limits.

3. State and prove Bolzano's Intermediate Value Theorem.

4. Prove that the function $f(x, y) = \sqrt{|xy|}$ is not differentiable at the origin but it is continuous at the origin, both fx, fy exist at the origin & have the value 0.

5. Apply Taylor's Theorem with Lagrange's form of remainder to the function $f(x) = \log x$ in $[1, x]$.

6. If $v_1 = x_1 + x_2 + x_3 + x_4$

$v_1 v_2 = x_2 + x_3 + x_4$

$v_1 v_2 v_3 = x_3 + x_4$

$v_1 v_2 v_3 v_4 = x_4$, show that $\frac{\partial(x_1, x_2, x_3, x_4)}{\partial(v_1, v_2, v_3, v_4)} = v_1^3 v_2^2 v_3$

SECTION-C

7. a) Show that the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{n+1}{n}\right) + \frac{2}{1} - \frac{3}{2} = \frac{4}{3} - \frac{5}{4} + \ldots\ldots$ oscillates finitely.

b) Use definition of limit to prove that $\lim_{x\to 3}(1 - 3x) = -8$.

8. a) Show that the function $f(x) = \frac{1}{\sqrt{x}}$ is differentiable at $x > 0$.

b) If $z = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$, $x \neq 0, y \neq 0$.

9. a) Prove that sequence $\left\{\frac{n}{n+1}\right\}$ is Cauchy sequence.

b) Prove that if f is continuous at $x = a$, the $|f|$ is also continuous at $x = a$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.