Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

Total No. of Questions : 09

Bachelor of Science - Honours (Mathematics) (Sem.–1)

CALCULAS-I

Subject Code : UC-BSHM-101-19

M.Code : 77312

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

l. Solve the following :

a) Find the *l.u.b.* and *g.l.b.,* if they exist for the set $A=\left\{\frac{2x+1}{x+5};|x-4|<2\right\}$.

b) Define the greatest integer function. Also write its domain and range.

c) Prove that $\frac{d}{dx}(\sinh x)=\cosh x$.

d) Differentiate $\cos^{-1}(2x^2-1)$ with respect to x if $0 < x < 1$.

e) Discuss the applicability of Rolle's Theorem for the function $f(x) = |x|$ in the interval $[-3, 3]$.

f) Evaluate $\lim_{x\to 1}\frac{\log x}{x-\sqrt{x}}$

g) Show that $y = x + a$ is the only asymptote of the curve $x^2(x-y) + ay^2 = 0$.

h) Find the nth derivative of $\frac{1}{(x+2)(x+3)}$.

i) Using $\in - \delta$ definition, prove that is continuous $f(x) = 3x + 2$ at $x = 2$.

j) State Cauchy's Mean Value theorem.

SECTION-B

2. a) State and prove Archimedean property of real numbers.

   b) Express the function $h(x)=\sqrt{\frac{x}{x-1}}$ as a composite of two 'simpler' functions, and state necessary conditions on their domains.

3. a) Prove that the function $f(x) = \frac{1}{x}$ is continuous in (0, 1) but is not uniformly continuous.

   b) Find all the asymptotes of the following curve :

$$x^3 - 4x^2y + 5xy^2 - 2y^3 + 3x^2 - 4xy + 2y^2 - 3x + 2y - 1 = 0$$

4. a) If $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, find $f'(x)$.

   b) If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$.

5. a) If $y = \log\left(x+\sqrt{x^2+a^2}\right)$, show that $\left(x^2+a^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.

   b) Find the derivative of $x^{\tanh x}$.

SECTION-C

6. a) Find the interval of concave upwards for the curve $y = (\cos x + \sin x)\, e^x$ in $(0, 2\pi)$.

   b) Show that the curve $x = \log\left(\frac{y}{x}\right)$ has a point of inflexion at $(-2, -2e^{-2})$.

7. a) Find the values of a and b, so that the $\lim_{x\to 0} \frac{x(1-a\cos x)+b\sin x}{x^3}$ exists and is equal to $\frac{1}{3}$.

b) Use Lagrange's Mean Value theorem to prove that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$ for $0 < x < 1$.

8. a) Find the nth derivative of $\sin x \sin 2x \sin 3x$.

b) If $y = (\sin^{-1} x)^2$, find $y_n(0)$

9. a) If $f(x) = \tan x$, then prove that

$$^{n}C_0 f^{n}(0) - {}^{n}C_2 f^{n-2}(0) + {}^{n}C_4 f^{n-4}(0) - \ldots\ldots\ldots = \sin \frac{n\pi}{2}.$$

b) Use Maclaurin's Theorem (with Lagrange's form of remainder) to expand $\sin x$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.