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Roll No. Total No. of Pages : 02

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Bachelor of Science - Honours (Mathematics) (Sem.–1)

CO-ORDINATE GEOMETRY

Subject Code : UC-BSHM-102-19

M.Code : 77313

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Solve the following :

a) For what values of k does the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0$ represent two straight lines?

b) Find the equations of the tangent and normal at the point of the parabola $y^2 = 8x$ whose ordinate is 4.

c) Find the pole of the line $x - 2y + 3 = 0$ w.r.t. the ellipse $3x^2 + 4y^2 = 12$.

d) If e_1 and e_2 be the eccentricities of a hyperbola and of the conjugate hyperbola, then show that $\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$.

e) Show that the equation $r^2 - 2br \cos(\theta - \alpha) = c$ describes a circle with centre (b, α) if $b^2 + c > 0$.

f) Show that if $ax^2 + 2hxy + by^2 = 1$ and $a' x^2 + 2h' xy + b' y^2 = 1$ represent the same conic and the axes are rectangular, then $(a - b)^2 + 4h^2 = (a' - b')^2 + 4h'^2$.

g) Find the equation to a circle, the axis of coordinates being two straight lines through its centre at right angles.

h) Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$.

i) Prove that the circles $x^2 + y^2 - 2ax + c = 0$ and $x^2 + y^2 + 2by - c = 0$ intersect orthogonally.

j) If pairs of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, prove that $pq = -1$.

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SECTION-B

2. Find the angle between the straight lines given by the equation $ax^2 + 2hxy + by^2 = 0$. Also find the condition of perpendicularity.

3. Find the equations of the lines joining the origin to the points of intersection of the line $2x - 3y + 4 = 0$ with the curve $x^2 + 4xy + 2y^2 + 12x + 4y = 0$, and show that they are at right angles.

4. a) Find the pole of the straight line $9x + y - 28 = 0$ with respect to the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$.

   b) Find the lengths of the tangents drawn to the circle $3x^2 + 3y^2 - 7x - 6y = 12$ from the point $(6, -7)$.

5. Find the locus of a point P which is such that its polar with respect to one circle touches a second circle.

SECTION-C

6. a) Prove that the locus of the poles of the chords which are normal to the parabola $y^2 = 4ax$ is the curve $y^2 (x + 2a) + 4a^3 = 0$.

   b) Prove that the chord of a parabola which subtends a right angle at the vertex meets its axis in a fixed point.

7. a) If the normal at the end of a latus rectum of an ellipse passes through one extremity of the minor axis, show that the eccentricity of the curve is given by the equation $e^4 + e^2 - 1 = 0$.

   b) The chord of contact of tangents from a point P to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ subtends a right angle at the centre. Prove that locus of P is the ellipse $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2} - \frac{1}{b^2}$.

8. a) Through what angle should the axes be rotated so that the mixed term may disappear from the equation $17x^2 - 16xy + 17y^2 - 225 = 0$?

   b) On shifting the origin to the point $(1, -1)$, the axis remaining parallel to the original axis, the equation of a curve becomes $4x^2 + y^2 + 3x - 4y + 2 = 0$. Find its original equation.

9. a) Identify the curve represented by the equation $3x^2 + 2xy + 3y^2 + 18x + 22y + 50 = 0$. Reduce it to standard form by suitable transformation of axes.

   b) Find the equation of the chord of contact of tangents drawn from the point (r_1, θ_1) to the conic $\frac{l}{r} = 1 - e\cos\theta$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

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