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Total No. of Questions : 09

B.Sc. (Hons) Aircraft Maintenance (2018 Batch) (Sem.–1)

MATHEMATICS

Subject Code : BSCARM-104-18

M.Code : 75635

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

SECTION-A

1. Do all the questions :

a) Define rank of a matrix. Find rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & -1\\ 1 & 3 & 2\end{bmatrix}$

b) State Cayley-Hamilton theorem and verify the same for $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$.

c) Prove that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A.\tan B}$.

d) Find sin 75° and cosec 75°.

e) Discuss continuity of $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$ at (0,0).

f) If $z=x^y+y^x$, verify that $\dfrac{\partial^2 z}{\partial x\partial y}=\dfrac{\partial^2 z}{\partial y\partial x}$.

g) Change the order of integration for $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} f(x,y)\,dxdy$.

h) Find the area bounded by the parabola $y^2=4ax$ and its latus rectum.

i) State Green's theorem in plane.

j) Find the divergence and curl of the vector field

$V = (x^2 - y^2)\,\hat{i} + 2xy\hat{j} + (y^2 - xy)\,\hat{k}$.

SECTION-B

2. Investigate for what values of λ and μ, the simultaneous equations

$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ has (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.

3. Show that $\cot^{-1} \dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} = \dfrac{\pi}{2} - \dfrac{x}{2}$, If $\dfrac{\pi}{2} < x < \pi$.

4. If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \text{ and } x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u.$$

5. Find the volume of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

6. If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, then using Jacobians show that $\left(\dfrac{\partial f}{\partial x}\right)^2 + \left(\dfrac{\partial f}{\partial y}\right)^2 = \left(\dfrac{\partial f}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial f}{\partial \theta}\right)^2$

SECTION-C

7. Examine whether the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. If so, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix.

8. A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.

9. State Stoke's theorem. Using Stoke's theorem evaluate $\int((x + y)dx + (2x - z)dy + (y + z)dz)$ over curve C where C is the boundary of the triangle with vertices (2,0,0), (0,1,0) and (0,0,6).

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.