Subject Code: R13102/R13 **Set No - 1**

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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PART-A

1.(i) Find the orthogonal trajectories of the curve $r = a(1 + \cos\theta)$.

(ii) If $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi, z = r\cos\theta$, find $\frac{\partial(r,\theta,\varphi)}{\partial(x,y,z)}$, given that $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2\sin\theta$.

(iii) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ using Heaviside function.

(iv) Let the heat conduction in a thin metallic bar of length L is governed by the equation $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$, t > 0. If both ends of the bar are held at constant temperature zero and the bar is initially has temperature f(x), find the temperature u(x,t).

(v) Solve $p^2 + pq = z^2$.

(vi) Find $\frac{1}{D^2-4D+4}x^2\sin x$. [4+4+4+4+3+3]

PART- B

2.(a) Solve $y(2x^2 - xy + 1)dx + (x - y)dy = 0$

(b) Find the complete solution of $y'' + 2y = x^2e^{3x} + e^x\cos 2x$ [8+8]

3.(a) Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

(b) Find the solution of $\frac{d^2y}{dx^2} + 4y = \operatorname{Sin} 3x + \cos 2x$. [8+8]

4.(a) Find the Laplace transform of $f(t) = \frac{\cos at - \cos bt}{t}$.

(b) If $x = \sqrt{vw}, y = \sqrt{uw}, z = \sqrt{uv}$ and
$u = r\sin\theta\cos\varphi,\ v = r\sin\theta\sin\varphi\ and\ w = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$. [8+8]

5.(a) Expand $f(x,y) = e^y\ln(1 + x)$ in powers of x and y using MacLaurin's Series

(b) Solve $y'' - 8y' + 15y = 9te^{2t},\ y(0) = 5\ and\ y'(0) = 10$ using Laplace transforms [8+8]

6.(a) Solve $(y + xz)p - (x + yz)q = x^2 - y^2$.

(b) Solve the partial differential equation px+qy =1. [8+8]

7.(a) Find the partial differential equation of all spheres whose centers lie on z- axis.

(b) Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$, if the initial deflection is

$$f(x) = \begin{cases} \frac{2k}{l}x & if\ 0 < x < l/2 \\ \frac{2k}{l}(l-x) & if\ \frac{l}{2} < x < l \end{cases}$$

and initial velocity equal to 0. [8+8]

Subject Code: R13102/R13 **Set No - 2**

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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PART-A

1.(i) Find the complete solution of $(D^4 + 16)y = 0$.

(ii) If $x = r\cos\theta, y = r\sin\theta, z = z$, find $\frac{\partial(r,\theta,z)}{\partial(x,y,z)}$, given that $\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = r$.

(iii) Solve $x^2p^2 + y^2q^2 = z^2$.

(iv) Find the solution, by Laplace transform method, of the integro-differential equation $y' + 3y + 2\int_0^t y(t)dt = t$

(v) Find the differential equation of the orthogonal trajectories for the family of parabola through the origin and foci on y-axis.

(vi) Find the solution of wave equation in one dimension using the method of separation of variables.

[3+3+4+4+4+4]

PART-B

2.(a) Solve $y(y^2 - 2x^2)dx + x(2y^2 - x^2)dy = 0$

(b) Find the complete solution of $y'' + 5y' - 6y = \sin 4x \sin x$.

[8+8]

3.(a) Solve $\cos x\, dy = y(\sin x - y)dx$.

(b) Find the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2xe^{3x} + 3e^x \cos 2x$.

[8+8]

4.(a) Find the Laplace transform of $f(t) = \int_0^t e^{-u}\cos u\, du$.

(b) Find the shortest distance from origin to the surface $xyz^2 = 2$.

[8+8]

5.(a) Find $\frac{\partial(u,v)}{\partial(r,\theta)}$ if $u = 2axy$ and $v = a(x^2 - y^2)$, where $x = r\cos\theta$ and $y = r\sin\theta$.

(b) Solve $y'' - 8y' + 15y = 9te^{2t}$, $y(0) = 5$ and $y'(0) = 10$ using Laplace transforms

[8+8]

6.(a) Form the partial differential equation by eliminating the arbitrary function from $xyz = f(x + y + z)$.

(b) Find the solution of $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$.

[8+8]

7.(a) Solve the partial differential equation $xzp + yzq = xy$.

(b) Find the temperature in a bar of length l which is perfectly insulated laterally and whose ends O and A are kept at 0°C, given that the initial temperature at any point P of the rod is given by f(x).

[8+8]

Subject Code: R13102/R13 **Set No - 3**

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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PART-A

1.(i) Find the dimensions of rectangular box of maximum capacity whose surface area is S.
(ii) Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$.
(iii) A generator having emf 100 volts is connected in series with a 10 ohm resistor and an inductor of 2 henries. If the switch is closed at a time t =0, find the current at time t>0.
(iv) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ using Heaviside function.
(v) Solve pq+qx = y.
(vi) Find the solution of $2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

[4+4+4+4+3+3]

PART- B

2.(a) Solve $y(1+xy)dx + x(1-xy)dy = 0$
(b) Find the complete solution of $y'' + 4y = e^x sin^2 x$.

[8+8]

3.(a) Solve $2x\,y' + y = \frac{2x^2}{y^3}, y(1) = 2$.
(b) Find the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = e^{2x} + 3\cos(4x+3)$.

[8+8]

4.(a) Find the Laplace transform of $f(t) = te^{-2t}cos\, t$.
(b) Find the maxima and minima of $x^3 + 3xy^2 - 15x^2 - 15\,y^2 + 72x$.

[8+8]

5.(a) Expand $f(x, y) = e^{xy}$ in powers of (x-1) and (y-1).
(b) Solve $y'' + 7y' + 10\,y = 4e^{-3t},\ y(0) = 0\ and\ y'(0) = -1$ using Laplace transforms.

[8+8]

6.(a) Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.
(b) Find the solution of $(4D^2 + 12DD' + 9D'^2)z = e^{3x-2y}$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$.

[8+8]

7.(a) Solve the partial differential equation $p\tan x + q\tan y = \tan z$.
(b) A tightly stretched string with fixed end points x=0 and x=l is initially in a position given by $y = y_0 sin^3\frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x,t)$.

[8+8]

Subject Code: R13102/R13 **Set No - 4**

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
*****

PART-A

1.(i) Find the distance from the centre at which the velocity in simple harmonic motion will be 1/3rd of the maximum.

(ii) Find a point with in a triangle such that the sum of the squares of its distances from the three vertices is minimum.

(iii) Find the solution, by Laplace transform method, of the integro-differential equation $y' + 4y = \int_0^t y(t)dt$, $y(0) = 0$.

(iv) Uranium disintegrates at a rate proportional to the amount present at that time. If M and N grams of Uranium that rae present at times T_1 and T_2 respectively, find the half life of Uranium.

(v) Find the complete solution of $(D^3 - 3D^2D' + 3\ DD'^2 - D'^3)z = 0$.

(vi) Solve $z^2 = 1 + p^2 + q^2$.

[4+4+4+4+3+3]

PART- B

2.(a) Solve $(3y^2 + 4xy - x)dx + x(x + 2y)dy = 0$

(b) Find the solution of $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = sin4x\ cosx$.

[8+8]

3.(a) Find the complete solution of $y'' + 2y = x^2e^{3x} + e^x \cos 2x$.

(b) Solve $x z' + z\log z = z(\log z)^2$.

[8+8]

4.(a) Find the Laplace transform of $f(t) = te^{2t}cos\ 2t$.

(b) If $u = sin^{-1}(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}})$, prove that $xu_x + yu_y = \frac{5}{2}\tan u$.

[8+8]

5.(a) If $w = (y - z)(z - x)(x - y)$, find the value of $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$.

(b) Solve $y'' + 2y' + 5\,y = e^{-t}\sin t$, $y(0) = 0\ and\ y'(0) = 1$ using Laplace transforms.

[8+8]

6.(a) Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $z = ax + by + a^2 + b^2$.

(b) Using method of separation of variables, solve $u_{xt} = e^{-t}cosx$ with $u(x,0) = u(0,t) = 0$.

[8+8]

7.(a) Find the temperature in a thin metal rod of length L, with both ends insulated and with initial temperature in the rod is $sin(\frac{\pi x}{L})$.

(b) Solve the partial differential equation $p\,x^2 + qy^2 = z^2$.

[8+8]