

# GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER- III EXAMINATION – SUMMER 2020

**Subject Code: 3130006**
**Date: 28/10/2020**
**Subject Name: PROBABILITY AND STATISTICS**
**Time: 02:30 PM TO 05:00 PM**
**Total Marks: 70**
**Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Statistical Tables are required.

- Q.1 (a)** An insurance company obtained the following data for accident claims (in thousand rupees) from a particular region. Find its mean, median and Mode. **03**

|           |       |       |       |       |        |         |
|-----------|-------|-------|-------|-------|--------|---------|
| Amount    | 1 – 3 | 3 – 5 | 5 – 7 | 7 – 9 | 9 – 11 | 11 – 13 |
| Frequency | 6     | 47    | 75    | 46    | 18     | 8       |

- (b)** A market survey was conducted in four cities to find out the preference for brand A soap. The responses are shown below: **04**

|            |       |         |         |        |
|------------|-------|---------|---------|--------|
|            | Delhi | Kolkata | Chennai | Mumbai |
| Yes        | 45    | 55      | 60      | 50     |
| No         | 35    | 45      | 35      | 45     |
| No opinion | 5     | 5       | 5       | 5      |

- (a) What is the probability that a consumer preferred brand A, given that he was from Chennai?

- (b) Given that a consumer preferred brand A, what is the probability that he was from Mumbai?

- (c) (i)** The number of monthly breakdowns of a computer is a random variable having Poisson distribution with mean 1.8. Find the probability that the computer will function for a month (a) without a breakdown (b) with at least one breakdown. **03**

- (ii)** Assume that 5 % of the apples weigh less than 150 gm and 20 % of the apples weigh more than 225 gm. If the distribution of the weight of the apples is normal, find the mean and standard deviation of the distribution. **04**

- Q.2 (a)** The probability that one of the ten telephone lines is busy at an instant is 0.2. **03**

- (a) What is the probability that 5 of the lines are busy?

- (b) What is the probability that all the lines are busy?

- (b)** An auto company claims that the mean petrol consumption of its new six cylinder car is 9.5 km per liter which is lower than the existing auto engine. It was found that the mean petrol consumption of a sample of 50 of these cars was 10 km per liter with a standard deviation of 3.5 km per liter. Test the claim at 5 % level of significance. **04**

- (c) (i)** The life of batteries manufactured by a battery manufacturer can be modelled as a random variable having approximately a normal distribution with  $\mu = 50$  months and  $\sigma = 6$  months. Find the probability that the mean of a random sample of 36 such batteries will be less than 48 months. **03**

- (ii)** If two random variables  $X$  and  $Y$  have the joint density **04**

$$f(x, y) = \begin{cases} k(x^2 + y), & \text{for } 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases},$$

find  $k$  and the mean of the conditional density  $f_1(x | 0.5)$  where  $f_1(x)$  is the marginal probability density of  $X$ .

**OR**

- (c) (i)** A process for making certain bearings is under control if the diameters of **03**

the bearings have a mean of 0.5000 cm. What can we say about this process if a sample of 10 of these bearings has a mean diameter of 0.5060 cm and a standard deviation of 0.0040 cm?

- (ii) Three balanced coins are tossed. Let  $X$  denote the number of heads on the first two coins and  $Y$  denote the number of tails on the last two coins. Find the joint distribution of  $X$  and  $Y$ . 04

- Q.3 (a)** Show that  $A$  and  $B$  are independent events if  $P(A) = 0.25$ ,  $P(B) = 0.40$ , and  $P(A \cup B) = 0.50$ . 03

- (b)** Given that  $n = 25$ ,  $\sum X = 125$ ,  $\sum X^2 = 650$ ,  $\sum Y = 100$ ,  $\sum Y^2 = 460$  and  $\sum XY = 508$ . Later on it was found that two of the points (8, 12) and (6, 8) were wrongly entered as (6, 14) and (8, 6). Prove that  $r = \frac{2}{3}$ . 04

- (c) (i)** The runs scored by two batsmen  $A$  and  $B$  in 10 matches are given in the following table: 03

|     |    |    |    |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|----|----|----|
| $A$ | 14 | 13 | 26 | 53 | 17 | 29 | 79 | 36 | 84 | 49 |
| $B$ | 37 | 22 | 56 | 52 | 14 | 10 | 37 | 48 | 20 | 4  |

Who is more consistent?

- (ii)** Calculate the first four moments about the mean of the following data: 04

|     |   |    |    |    |    |
|-----|---|----|----|----|----|
| $x$ | 5 | 10 | 15 | 20 | 25 |
| $f$ | 6 | 10 | 14 | 6  | 4  |

**OR**

- Q.3 (a)** The number of page requests that arrive at a Web server is a Poisson random variable. Its probability distribution is as follows: 03

|                             |       |       |       |       |       |       |       |
|-----------------------------|-------|-------|-------|-------|-------|-------|-------|
| Number of requests/sec. $x$ | 0     | 1     | 2     | 3     | 4     | 5     | 6     |
| Probability $f(x)$          | 0.368 | 0.368 | 0.184 | 0.061 | 0.015 | 0.003 | 0.001 |

Find the mean and variance of this probability distribution.

- (b)** From the following data of the marks obtained by 8 students in Computer Networking (CN) and Compiler Design (CD) papers, compute rank coefficient of correlation. 04

|    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|
| CN | 15 | 20 | 28 | 12 | 40 | 60 | 20 | 80 |
| CD | 40 | 30 | 50 | 30 | 20 | 10 | 30 | 60 |

- (c) (i)** Find out mean deviation about median for the following series: 03

|       |   |   |   |    |    |    |    |
|-------|---|---|---|----|----|----|----|
| Size  | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| Freq. | 1 | 2 | 4 | 5  | 4  | 3  | 1  |

- (ii)** Find the coefficient of skewness based on the Method of Moments for the following data: 04

|       |        |         |         |         |         |
|-------|--------|---------|---------|---------|---------|
| Class | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
| $f$   | 13     | 20      | 30      | 25      | 12      |

- Q.4 (a)** In a certain district, the proportion of highway sections requiring repairs in any given year is a random variable having the probability density 03

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the distribution function and use it to determine the probability that at least half of the highways sections will require repairs in any given year.

- (b)** At a checkout counter customers arrive at an average of 1.5 per minute. Find the probabilities that (i) at most 4 will arrive in any given minute; (ii) one customer will arrive in the first one minute and two customers will arrive in the next one minute. 04

- (c)** Fit a parabola  $y = a + bx + cx^2$  to the following data: 07

|     |   |   |    |    |    |
|-----|---|---|----|----|----|
| $x$ | 0 | 1 | 2  | 3  | 4  |
| $y$ | 1 | 4 | 10 | 17 | 30 |

**OR**

- Q.4 (a)** The joint probability density of two random variables  $X_1$  and  $X_2$  is given by 03

$$f(x_1, x_2) = \begin{cases} \frac{1}{5}(x_1 + 2x_2), & \text{for } 0 < x_1 < 1, 0 < x_2 < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal densities of both the random variables and check whether the two random variables are independent.

- (b) A safety engineer feels that 30 % of all industrial accidents in her plant are caused by failure of employees to follow instructions. If this figure is correct, find approximately, the probability that among 84 industrialized accidents in this plant anywhere from 20 to 30 (inclusive) will be due to failure of employees to follow instructions. **04**
- (c) The following show the gain in reading speed of 3 students in a speed-reading program, and the number of weeks they have been in the program: **07**

|              |    |     |    |     |     |     |    |     |
|--------------|----|-----|----|-----|-----|-----|----|-----|
| No. of weeks | 3  | 5   | 2  | 8   | 6   | 9   | 3  | 4   |
| Speed gain   | 86 | 118 | 49 | 193 | 164 | 232 | 73 | 109 |

Fit a straight line by the method of least squares.

- Q.5** (a) Suppose that the time it takes to get service in a restaurant follows a gamma distribution with mean 8 minutes and variance 32 minutes. Suppose that you went to this restaurant at 6: 30 p.m. What is the probability that you will receive service before 6: 36 p.m.? **03**
- (b) If 57 out of 150 patients suffering from certain disease are cured by allopathy and 33 out of 100 patients with the same disease are cured by homeopathy, is there reason to believe that allopathy is better than homeopathy at 0.05 level of significance? **04**
- (c) (i) If two independent random samples of size  $n_1 = 7$  and  $n_2 = 13$  are taken from a normal population, what is the probability that the variance of the first sample will be at least three times as large as that of the second sample? **03**
- (ii) A courier service advertises that its average delivery time is less than 5 hours for local deliveries. A random sample of 10 for the amount of time this courier service takes to deliver packages to an addressee across town produced the following times: 8, 3, 4, 7, 10, 5, 6, 4, 5, 8. Is this evidence support the claim of the courier service at 5 % level of significance? **04**

**OR**

- Q.5** (a) A power supply unit for a computer component is assumed to follow an exponential distribution with a mean life of 1200 hours. What is the probability that the component will survive more than 1500 hours? **03**
- (b) Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate if attacked by this disease is 85 % in favour of the hypothesis that it is more at 5 % level. **04**
- (c) (i) The mean life of a random sample of 10 light bulbs was found to be 1456 hours with a standard deviation of 423 hours. A second sample of 17 bulbs chosen at random from a different batch showed a mean life of 1280 hours with a standard deviation of 398 hours. Is the difference between the mean life of the two batches significant at 5 % level of significance? **03**
- (ii) The manager of a theatre complex with four theaters wanted to see whether there was difference in popularity of the four movies currently showing for Saturday afternoon with the following results: 86, 77, 84, 81 customers viewed movies 1, 2, 3, and 4 respectively. Complete the test to see whether there is a difference at the 5 % level of significance. **04**

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