Seat No.: Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III EXAMINATION – SUMMER 2020

Subject Code: 3130107 **Date:27/10/2020**

Subject Name: Partial Differential Equations and Numerical Methods

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Use Iteration method to find the real root of the equation $x^3 + x - 1 = 0$ correct to six decimal places starting with $x_0 = 1$. **03**

(b) Use Bisection method to find the real root of the equation $x - \cos x = 0$ correct upto four decimal places. **04**

(c) Explain the Newton-Raphson method briefly. Also find an iterative formula for $\sqrt{N}$ and hence find $\sqrt{7}$ correct to three decimal places. **07**

Q.2 (a) Evaluate $\int_0^1 e^{x^2} dx$ by Simpson's-1/3 rule with n=10 and estimate the error. **03**

(b) Solve the following linear system of equations by Gauss elimination method. **04**

$0 \bullet x + 8y + 2z = -7$

$3x + 5y + 2z = 8$

$6x + 2y + 8z = 26$

(c) Compute $\cosh(0.56)$ using Newton's forward difference formula and also estimate the error for the following table. **07**

x	0.5	0.6	0.7	0.8
$f(x)$	1.127626	1.185465	1.255169	1.337435

OR

(c) The speed, v meters per second, of a car, t seconds after it starts, is show in the following table. **07**

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	4.50	4.5	5.4	9.0

Using Simpson's $1/3$ rule, find the distance travelled by the car in 2 minutes.

Q.3 (a) Use Trapezoidal rule to evaluate $\int_0^1 x^3 dx$ using five subintervals. **03**

(b) Check whether the following system is diagonally dominant or not. If not, rearrange the system and solve it using Gauss-Seidel method. **04**

$8x - 3y + 2z = 20$

$4x - 11y - z = 33$

$6x - 3y + 12z = 35$

(c) Explain Euler's method briefly and apply it to the following initial value problem by choosing $h = 0.2$ and hence obtain $y(1.0)$. $\frac{dy}{dx} = x + y \,\middle|\, y(0) = 0$. Also determine the error by deriving it analytical solution. **07**

OR

Q.3 **(a)** Use Runge-Kutta second order method to find the approximate value of $y(0.2)$ given that $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$ & $h = 0.1$ **03**

(b) Find the Lagrange interpolating polynomial from the following data **04**

x	0	1	4	5
$f(x)$	1	3	24	39

(c) Derive Secant iterative method from the Newton-Raphson method and use it to find the root of the equation $\cos x - xe^x = 0$ correct to four decimal places. **07**

Q.4 **(a)** $(x^2 - yz)p + (y^2 - xy)q = z^2 - xy$. **03**

(b) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$. **04**

(c) Obtain the solution of following one-dimensional Wave equation together with following initial and boundary conditions by the method of separation of variables. **07**

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$
$$u(0,t) = u(l,t) = 0 \ \forall t > 0$$
$$u(x,0) = f(x) \text{ for } 0 < x < l$$
$$u_t(x,0) = g(x) \text{ for } 0 < x < l$$

OR

Q.4 **(a)** $pz - qz = z^2 + (x+y)^2$. **03**

(b) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$. **04**

(c) Obtain the solution of following one-dimensional heat equation with insulated sides by the method of separation of variables. **07**

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$
$$u(0,t) = u(l,t) = 0 \ \forall t > 0$$
$$u(x,0) = f(x) \text{ for } 0 < x < l$$

Q.5 **(a)** $p^2 - q^2 = x - y$ **03**

(b) Solve the given equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given that $u(x,0) = 6e^{-3x}$. **04**

(c) Obtain the solution of following one-dimensional heat equation with insulated ends by the method of separation of variables. **07**

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$
$$u_x(0,t) = u_x(l,t) = 0 \ \forall t > 0$$
$$u(x,0) = f(x) \text{ for } 0 < x < l$$

OR

Q.5 **(a)** $z^2(p^2 + q^2 + 1) = a^2$, **03**

(b) Using method least squares, find the best fit straight line for the following data. **04**

x	1	2	3	4	5
y	1	3	5	6	5

(c) Obtain the solution following two-dimensional Laplace equation. **07**

$$u_{xx} + u_{yy} = 0$$
$$u(x,0) = u(x,\infty) = u(a,y) = 0$$
$$u(x,0) = f(x)$$

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