Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE – SEMESTER 1&2 EXAMINATION – SUMMER 2020

Subject Code: 3110015 **Date:09/11/2020**

Subject Name: Mathematics II

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Evaluate $\int_c \bar{F}\cdot d\bar{r}$ along the parabola $y^2 = x$ between the points (0, 0) and (1, 1) where $\bar{F} = x^2\hat{i} + xy\,\hat{j}$	**03**
	(b)	Find the work done in moving particle from A (1, 0, 1) to B (2,1,2) along the straight-line AB in the force field $\bar{F} = x^2\hat{i} + (x-y)\,\hat{j} + (y+z)\hat{k}$	**04**
	(c)	Verify green`s theorem for $\int_c (2xydx - y^2dy)$ where C is the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$	**07**
Q.2	**(a)**	Find the Laplace transform of $te^{-4t}\sin 3t$.	**03**
	(b)	Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$.	**04**
	(c)	Show that the vector field $\bar{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\,\hat{j} + (xy\cos z + y^2)\hat{k}$ is conservative and find the corresponding scalar potential.	**07**
		OR	
	(c)	Show that $\bar{F} = 2xyz\,\hat{i} + (x^2z + 2y)\,\hat{j} + x^2y\,\hat{k}$ is irrotational and find a scalar function ϕ such that $\bar{F} = grad\phi$.	**07**
Q.3	**(a)**	Find the directional derivative of $f(x,y) = xy + xe^y + \cos(xy)$ at the point $P(1,0)$ in the direction of $\bar{u} = 3\hat{i} - 4\hat{j}$.	**03**
	(b)	Find the inverse Laplace transform of $\log\left(1+\frac{1}{s^2}\right)$.	**04**
	(c)	Find the singular solution and general solution of $y + px = x^4p^2$	**07**
		OR	
Q.3	**(a)**	Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.	**03**
	(b)	Show that $\int_0^\infty \frac{\omega^3\sin\omega x}{\omega^4+4}\,d\omega = \frac{\pi}{2}e^{-x}\cos x;\ x>0.$	**04**
	(c)	Find the power series solution of $y' - 2xy = 0;\ y(0) = 1$ near $x = 0$.	**07**

Q.4 **(a)** Find the Laplace transform of $e^{-t}\{1-u(t-2)\}$. **03**

(b) Solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2, \frac{dx}{dt} = -1$ at $t = 0$. **04**

(c) Solve $(D^2 - 1)y = xe^x \sin x$ **07**

OR

Q.4 **(a)** Solve $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$ **03**

(b) Using method of variation of parameter, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. **04**

(c) Using method of undetermined coefficients solve **07**
$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^x$.

Q.5 **(a)** Classify the singular points of $x^2y'' + xy' - 2y = 0$. **03**

(b) Solve $\frac{d^2y}{dx^2} + 9y = \sin 2x \sin x$. **04**

(c) Solve (i) $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$. **07**

(ii) $\frac{dy}{dx} + y\cot x = 2\cos x$.

OR

Q.5 **(a)** Solve $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$. **03**

(b) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = \cos(\ln x)$. **04**

(c) Using Frobenius method solve $2x^2y'' + xy' - (x+1)y = 0$. **07**

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