GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER– IV EXAMINATION – SUMMER 2020

Subject Code: 2141905   **Date:28/10/2020**

Subject Name: COMPLEX VARIABLES AND NUMERICAL METHODS

Time: 10:30 AM TO 01:30 PM   **Total Marks: 70**

Instructions:
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

                                                      MARKS

Q.1 **(a)** Verify Cauchy-Riemann equation for $f(z) = \cos x \cosh y - i \sin x \sinh y$.   **03**

(b) Find all cube roots of complex number $(-8i)$.   **04**

(c) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ with $h = 1$ by (i) Trapezoidal Rule (ii) Simpson's $\frac{1}{3}$ Rule (iii) Simpson's $\frac{3}{8}$ Rule.   **07**

Q.2 **(a)** Find the principal value of $(1-i)^{4i}$.   **03**

(b) Using Parametric representation of C, evaluate $\int_C \frac{z+2}{z} dz$; C is the circle $z = 2e^{i\theta} \left(0 \le \theta \le 2\pi \right)$ **04**

(c) Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic function and find harmonic conjugate $v(x,y)$.   **07**

OR

(c) For $f(z) = \begin{cases} \dfrac{(\bar{z})^2}{z}; & z \ne 0 \\ 0 & ; & z = 0 \end{cases}$,   **07**

Show that C-R equation is satisfied at origin but $f'(0)$ does not exist.

Q.3 **(a)** Derive the Taylor series representation

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}; \quad |z-i| < \sqrt{2}.$$ **03**

(b) State Cauchy Integral formula. Use it to evaluate $\int_C \frac{\cos z}{z(z^2+8)} dz$; C: $|z| = 1$.   **04**

(c) Find Laurent series representation of $f(z) = \dfrac{1}{z(1+z^2)}$

for (i) $0 < |z| < 1$ (ii) $1 < |z| < \infty$.   **07**

OR

Q.3 **(a)** Determine residue of $f(z) = \dfrac{3z^3 + 2}{z^2 + 9}$ at $z = 3i$.   **03**

(b) Find the fixed points of the transformation $w = \dfrac{z-1}{z+1}$.   **04**

                                                                1

(c) Evaluate $\int_0^\infty \dfrac{x^2 dx}{\left(x^2+1\right)\left(x^2+4\right)}$ using residues. **07**

Q.4 **(a)** Show that $1+\Delta \equiv e^{hD}$ **03**

(b) Find $f(0.12)$ & $f(0.26)$ by appropriate interpolation formula from following table **04**

x	0.10	0.15	0.20	0.25	0.30
$f(x)$	0.1003	0.1511	0.2027	0.2553	0.3093

(c) Determine images of Vertical and Horizontal lines under the transformation $w = e^z$. **07**

OR

Q.4 **(a)** Using Lagrange's formula, express the function $\dfrac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$ as a sum of partial fractions. **03**

(b) Find interpolating polynomial using Newton's divided difference formula from following table **04**

x	0	1	4	5	7
$f(x)$	−6	−3	138	369	1611

(c) Show that a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D if and only if v is a harmonic conjugate of u. **07**

Q.5 **(a)** Use Newton-Raphson method to find positive root of $\sin x = 1 - x$ correct to three decimal places. **03**

(b) Use power method to find largest eigen value and corresponding eigen vector of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ correct to four decimal places. **04**

(c) Apply Runge-Kutta fourth-order method to find $y(0.2)$. Given that $dy/dx = y - x$ where $y(0) = 2$ and $h = 0.1$. **07**

OR

Q.5 **(a)** Use Secant method to find a positive root of the equation $x^3 + x - 1 = 0$ correct to three decimal places. **03**

(b) Given that $\dfrac{dy}{dx} = x^2 + y$; $y(0) = 1$. Find $y(0.1)$ using Modified Euler's method with $h = 0.05$ correct to three decimal places. **04**

(c) Solve the following liner system **07**

$10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10z = 22$

Correct to two decimal places by Gauss-Seidel method.

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