

## GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER- III EXAMINATION – SUMMER 2020

Subject Code: 130001

Date: 26/10/2020

Subject Name: MATHEMATICS-III

Time: 02:30 PM TO 05:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) (i) Solve  $y' + \frac{1}{x^2} y = e^{\frac{1}{x}}$  03  
(ii) Solve  $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$  04
- (b) Find the power series solution of the equation  $\frac{d^2y}{dx^2} + xy = 0$ . 07
- Q.2** (a) (i) Solve  $y'' - 5y' + 6y = e^{4x}$  03  
(ii) Using the method of variation of parameter, solve  $y'' - 4y' + 4y = \frac{e^{2x}}{x}$  04
- (b) Using the method of undetermined coefficient, solve  $y''' + 3y'' + 2y' = x^2 + 4x + 8$  07
- OR**
- (b) Solve the equation by series method  $(x-2)y'' - x^2y' + 9y = 0$  about  $x = 0$ . 07
- Q.3** (a) Find the Fourier series of  $f(x) = x + x^2$  in the interval  $(-\pi, \pi)$ . Hence, deduce that  $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  07
- (b) Find the Fourier series of  $f(x) = e^{-x}$ ,  $-a < x < a$ . 07
- OR**
- Q.3** (a) Find the Fourier series of  $f(x) = |x|$ ,  $-\pi < x < \pi$ . 07
- (b) Find the Half range Fourier cosine series of  $f(x) = x \sin x$ ,  $0 \leq x \leq \pi$ .  $f(x+2\pi) = f(x)$  07
- Q.4** (a) (1) Find the Laplace transform of the function  $f(t) = e^{3t} \sin 2t$ . 03  
(2) Find the inverse Laplace transform of the function  $F(s) = \frac{s+7}{s^2 + 8s + 25}$ . 04
- (b) Solve the differential equation using Laplace Transformation method  $\frac{d^2y}{dt^2} + y = \sin t$ , Given that  $y(0) = 1$ ,  $y'(0) = 0$ ,  $t > 0$ . 07
- OR**
- Q.4** (a) (1) Find the Laplace transform of the function  $f(t) = t \cos^2 t$  03  
(2) Find the inverse Laplace transform of the function  $F(s) = \frac{6s-4}{s^2 - 4s + 20}$  04
- (b) Define Convolution theorem for Laplace transform. Using Convolution theorem to find Laplace inverse of the function  $F(s) = \frac{1}{(s^2 + a^2)^2}$  07

- Q.5** (a) (i) Form the partial differential equation of  $f(x^2 - y^2, x y z) = 0$ . **03**  
 (ii) Solve  $(y + z)p + (x + z)q = x + y$ . **04**

- (b) Solve by the method of separation of variables  $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$  **07**

**OR**

- Q.5** (a) (i) Solve  $p^2 + q^2 = x + y$  **03**  
 (ii) Solve  $pyz - zxq = xy$  **04**

- (b) Find the Fourier integral of the function  $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$

Hence, evaluate (i)  $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$  (ii)  $\int_0^\infty \frac{\sin \lambda}{\lambda} d\lambda$  **07**

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