Seat No.: _____ Enrolment No.

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2020

Subject Code:2140001 **Date:09/02/2021**

Subject Name:Mathematics-4

Time:02:30 PM TO 04:30 PM **Total Marks:47**

Instructions:

1. **Attempt any THREE questions from Q.1 to Q.6.**
2. **Q.7 is compulsory.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Define following terms : (a) Analytic function (b) continuous function	**03**
	(b)	Determine the bilinear transformation which maps the points $z = 2, 1, 0$ into the points $w = 1, 0, i$ respectively.	**04**
	(c)	Use Gauss-elimination method (with Partial Pivoting) to obtain the solution of the system $2x + 2y + z = 6$, $4x + 2y + 3z = 4$, $x + y + z = 0$	**07**
Q.2	**(a)**	Using the C-R equations, show that $f(z) = z^3$ is analytic everywhere.	**03**
	(b)	Evaluate $\int_C \frac{5z-2}{z(z-1)}\,dz$, where C is the circle $\lvert z\rvert = 3$.	**04**
	(c)	Show that $u(x, y) = x^2 - y^2$ is Harmonic. Find the corresponding analytic function $f(z) = u + iv$.	**07**
Q.3	**(a)**	Expand $f(z) = e^z$ in a Taylor series about $z = 0$.	**03**
	(b)	Determine the residues of $f(z) = \frac{(z-3)}{(z+1)(z+2)}$ at each of its poles in the finite z plane.	**04**
	(c)	Determine the Laurent series expansion of $f(z) = \frac{1}{(z-1)} - \frac{1}{(z-2)}$ valid for (a) $\lvert z\rvert < 1$ (b) $1 < \lvert z\rvert < 2$	**07**
Q.4	**(a)**	Check whether the function $f(z) = \bar{z} + 1$ is analytic or not at any point.	**03**
	(b)	Find the radius of convergence of the $\sum_{n=0}^{\infty} \frac{z^n}{n!}$	**04**
	(c)	Using Residue theorem, evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)}\,dz$ where C is circle $\lvert z\rvert = 3$	**07**
Q.5	**(a)**	Perform five iterations of Bisection method to find the real root of equation $x^3 - x - 1 = 0$.	**03**
	(b)	Solve the given System of Linear equations by using Gauss Elimination method: $x+3y+2z=5$, $2x+4y-6z=-4$, $x+5y+3z=10$	**04**
	(c)	Use second order Runge-Kutta method to solve $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ and find $y(0.2)$ with $h = 0.1$	**07**

Q.6 **(a)** Perform three iteration of secant method to find approximate root of equation $x^3+x^2-3x-3=0$. **03**

(b) Use Euler's method to solve $\frac{dy}{dx}=x+2y$, $y(1)=1$. Hence find $y(1.5)$ with $h=0.1$. **04**

(c) Using Lagrange's interpolating polynomial, find $f(10)$ from the given data: **07**

x	5	6	9	11
$f(x)$	12	13	14	16

Q.7 Find a real root of $x^3+x-1=0$, correct to two decimal places using Newton-Raphson method. **05**

OR

Construct an Interpolating polynomial which takes the following values : **05**

x	1	2	7	8
y	1	5	5	4

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