

GUJARAT TECHNOLOGICAL UNIVERSITY
BE- SEMESTER-IV (NEW) EXAMINATION – WINTER 2020
Subject Code:2141905
Date:09/02/2021
Subject Name:Complex Variables and Numerical Methods
Time:02:30 PM TO 04:30 PM
Total Marks:47
Instructions:

1. Attempt any THREE questions from Q.1 to Q.6.
2. Q.7 is compulsory.
3. Make suitable assumptions wherever necessary.
4. Figures to the right indicate full marks.

MARKS

Q.1 (a) Separate real and imaginary parts of $f(z) = e^{(z+2)}$, and also prove that it is analytic everywhere. **03**

(b) Use De Moivre's theorem and find 4th root of unity in the complex plane. **04**

(c) Use Gauss-Jacobi method to determine roots of the following equations **07**
 $20x + y - 2z = 17$
 $3x + 20y - z = -18$
 $2x - 3y + 20z = 25$

Q.2 (a) Evaluate the following integral along the curve **03**
$$\int_0^{2+4i} \operatorname{Re}(z) dz$$

$$z(t) = t + it^2$$

(b) Evaluate $\oint_C \frac{\cos \pi z}{z-1} dz$ where C is the circle **04**
1) $|z| = 2$ 2) $|z| = 1/2$

(c) Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find its conjugate harmonic function v . **07**

Q.3 (a) Obtain the Taylor's series of $f(z) = \sin z$ in powers of $(z - \frac{\pi}{4})$. **03**

(b) Find the center and radius of convergence of the power series $\sum_{n=0}^{\infty} (n+2i)^n z^n$. **04**

(c) Find the Laurent's series expansion of $f(z) = \frac{1}{(z+1)(z-2)}$ in the region **07**
1) $1 < |z| < 2$ 2) $|z| > 2$

Q.4 (a) Find the Maclaurin's series of $f(z) = \sin^2 z$ **03**

(b) Find all values of z such that $e^z = 1 + i$ **04**

(c) Evaluate $\oint_C \frac{\cos z}{z^2-4} dz$ counterclockwise around C: $|z| = 5/2$ **07**

Q.5 (a) Use Bisection method to find the real root of $x^3 - 4x - 9 = 0$. (Do 4 iterations) **03**

(b) Using Newton's divided difference interpolation formula, compute $f(10.5)$ from the following data: **04**

x	10	11	13	17
f(x)	2.3026	2.3979	2.5649	2.8332

(c) Use Simpson's 3/8 rule and evaluate the following integral taking $n=6$ and hence calculate $\log_e 2$. Also, find the error involved in the process. 07

$$\int_0^3 \frac{dx}{1+x}$$

Q.6 (a) Approximate the root of the equation $e^x - 2 \cos x = 0$, by three iterations of Newton Raphson method, taking initial approximation as $x_0 = 2$. 03

(b) Find an approximate value of $f(3.6)$ using Newton's backward difference formula from the following data: 04

x	0	1	2	3	4
$f(x)$	-5	1	9	25	55

(c) Using power method, determine the largest eigenvalue and the corresponding eigenvector of the matrix $= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, taking 07

initial eigenvector $x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

Q.7 Using three point Gaussian formula evaluate the following integral and compare with the exact value. 05

$$\int_{-1}^1 \frac{dx}{1+x^2}$$

OR

Q.7 Solve the following system of linear equations using Gauss Elimination Method. 05

$$x + y + z = 9; 2x - 3y + 4z = 13; 3x + 4y + 5z = 40$$
