

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER-VI (NEW) EXAMINATION – WINTER 2020

Subject Code:2160704

Date:27/01/2021

Subject Name:Theory of Computation

Time:02:00 PM TO 04:00 PM

Total Marks: 56

Instructions:

1. Attempt any FOUR questions out of EIGHT questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a) Discuss Recursive definition. Also define the language L defined by the following recursive definition over $\Sigma = \{a, b\}$: 03

 $\Lambda \in L$;For every $x \in L$, xa , bx , and abx are in L;

Nothing else is in L.

- (b) Let relation $R = \{(a,b) : a + b = 10 \text{ and } a, b \in \mathbb{N}\}$. Decide whether R is an equivalence relation or not. Justify your answer with proper reason. 04

- (c) Using the principle of mathematical induction, for all $n > 0$, prove that, 07

$$1 \times 2 + 3 \times 4 + 5 \times 6 + \dots + (2n - 1) \times 2n = \frac{n(n+1)(4n-1)}{3}$$

- Q.2 (a) Write regular expressions for the following languages defined over $\Sigma = \{0, 1\}$: 03

(i) The language of all the strings that do not end with 01.

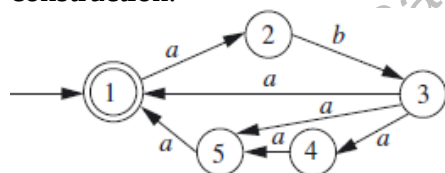
(ii) The language of all the strings containing even number of 0's and even number of 1's.

- (b) Draw DFA for the following languages defined over $\Sigma = \{a, b\}$: 04

(i) The language of all the strings with next-to-last symbol is a.

(ii) The language of all the strings containing substring bba.

- (c) Convert the following NFA into its equivalent DFA using the subset construction. 07



- Q.3 (a) Prove that the context-free languages are closed under union. 03

- (b) For the following CFG, find out two left most derivations for the string "aaabb" and also draw the corresponding parse trees. 04

 $S \rightarrow XY$ $X \rightarrow XX \mid a$ $Y \rightarrow YY \mid b$

- (c) Define CNF. Also convert the following CFG into its equivalent CNF. 07

 $S \rightarrow aX \mid Y \mid bab$ $X \rightarrow \wedge \mid Y$ $Y \rightarrow bb \mid bXb$

- Q.4 (a) What language over $\{a, b\}^*$ does the CFG with productions 03

 $S \rightarrow aT \mid bT$ $T \rightarrow aS \mid bS \mid \wedge$

generate? Prove your answer.

- (b) Let M_1 and M_2 be the FAs pictured in Fig. (i) and Fig. (ii) accept the languages L_1 and L_2 , respectively. 04

Fig. (i)

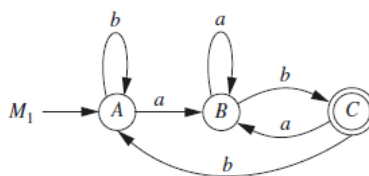
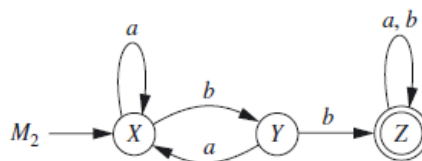


Fig. (ii)



Draw FAs accepting the following languages:

- (i) $L_1 \cup L_2$
 - (ii) L'_2
 - (c) Find context-free grammar generating the languages below. 07
 - (i) $\{a^i b^j c^k \mid j = i \text{ or } j = k\}$
 - (ii) $\{a^i b^j c^k \mid j \neq i + k\}$
- Q.5**
- (a) Define - A Pushdown Automaton and acceptance by a PDA. 03
 - (b) Convert the CFG with following productions into its equivalent PDA. 04
 $S \rightarrow [S] \mid SS \mid \wedge$
 - (c) Design a PDA to accept $L = \{wcw^R \mid w \in (a,b)^*\}$. 07
- Q.6**
- (a) Discuss pumping lemma for context free languages. 03
 - (b) Define bijection. Decide and justify whether the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(n) = n^2$ is bijection or not. 04
 - (c) Design a PDA to accept $L = \{xycy \mid x, y \in (a,b)^* \text{ and } |x| = |y|\}$. 07
- Q.7**
- (a) Discuss - recursively enumerable languages. 03
 - (b) Discuss - universal Turing machine. 04
 - (c) Draw Turing machine for $L = \{xx \mid x \in \{a, b\}^*\}$. Also trace out the same on input string aba. 07
- Q.8**
- (a) Discuss chomsky hierarchy. 03
 - (b) Discuss primitive recursive function using proper example. 04
 - (c) Draw Turing machine to accept language $L = \{x \in \{a, b\}^* \mid x \text{ ends with aba}\}$. Also trace out the same on input string aba. 07
