**Duration 3 hours** **Total 100 marks**

N.B: 1) Question No. 1 is compulsory.
2) Attempt any four out of remaining six questions.
3) Figures to the right indicate full marks.

1. (a) Let A={3,5,9,15,24,45} and relation R be defined on B by ${}_xR_y$ if and only if **(10)**
"x divides y". Show that R is a partial order relation
1.Draw the diagraph and Hasse diagram of R
2. Determine all minimal & all maximal elements.
3. find all least and greatest elements.
4. Give upper bounds and LUB of A={3,5)
5. Give all lower bounds and the GLB = {15,45}

(b) (i) Establish the following result using truth tables. **(05)**
(P ^ Q) ↔(¬RvQ) v P

(ii) What is the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$, with initial condition $a_0 = 2$, $a_1 = 7$ **(05)**

2. (a) (i) Write converse , inverse and contra positive of the following statement. **(05)**
" If weather will not be good then I will not travel."

(ii) Obtain the disjunctive normal form of (P->Q)^(¬P^Q) **(05)**

(b) (i) Find $\Delta a_n$ where $a_n = n^2 + n + 1$ where Δ denotes forward difference. **(05)**

**(05)**

(ii) For the set A = {a,b,c} give all the permutations of A. Show that the set of all permutations of A is a group under the composition operation.

3. (a) Obtain the recurrence relation and initial conditions to find the maximum number of regions defined by n lines in a plane. Assume that the lines are not parallel and lines not intersecting at one point when n>2. Solve the recurrence relation. **(10)**

(b) (i) Draw the transition state diagram of the finite state machine $M=(S,I,O,\delta,\lambda,s_0)$ given in the table **(05)**

| | δ | | λ | |
|---|---|---|---|---|
| | a | b | *a* | *b* |
| $S_0$ | $S_1$ | $S_2$ | x | y |
| $S_1$ | $S_3$ | $S_1$ | y | z |
| $S_2$ | $S_1$ | $S_0$ | z | x |
| $S_3$ | $S_0$ | $S_2$ | z | x |

(ii) Explain with suitable example:- (1) Predicate (2) Proposition **(05)**

4. (a) Determine whether the relation R on a set A is reflective ,irreflective, asymmetric, antisymmetric or transitive. **(10)**
A = set of all positive integers, ${}_aR_b$ iff a≤b+1

(b) (i) Show by mathematical induction, that for all $n \geq 1$, **(05)**
$1+5+9+\text{---------------}+(4n-3) = n(2n-1)$

(ii) Let G be a group. Show that the function $f:G \rightarrow G$ defined by **(05)**
$f(a) = a^2$ is a homomorphism iff G is abelian.

5. (a) (i) Let T be set of even integers. Show that the semigroups (Z,+) and **(05)**
(T,+) are Isomorphic, where Z is a set of integers.

(ii) For the grammar specified below describe precisely the **(05)**
language,L(G),produced. Also give the corresponding syntax
diagram for the productions of the grammar. $G=(V,S,v_0,|\rightarrow)$
$V = \{v_0,a,b\}$, $S = \{a,b\}$
$v_0|\rightarrow aav_0$ , $v_0|\text{-->}$ a, $v_0 |\rightarrow b$

(b) (i) perform the following **(10)**
i) $0111 \times 1010 = ?$
ii) $(413)_8 = (?)_{10}$
iii) $10100 \div 100 = ?$
iv) $(1101)_2 - (1001)_2 = ?$
v) $(49.25)_{10} = (?)_2$

6. (a) (i) Determine the validity of the following argument using deduction **(05)**
method:
" If I study then I will pass examination . If I do not go to picnic ,then I will study. But I failed examination. Therefore , I went to picnic"

(ii) Let G be a group and let 'a' be a fixed element of G. show that the **(05)**
function $f_a:G\rightarrow G$ defined by $f_a(x) = axa^{-1}$ for $x \in G$ is an
isomorphism.

(b) (i) **(05)**

Let $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix.

Determine the group code $e_H$: $B^2\text{-->}B^5$. How many errors will the
above group code detect.

(ii) Let A={1,2,3,4}. For the relation
R={(1,1),(1,4),(2,2),(3,3),(2,1),(4,4) find the matrix of transitive **(05)**
closure by using Warshall's algorithm.

7. (a) Show that (2,5) encoding function $e:B^2\text{-->} B^5$ defined **(10)**
bye(00)=00000,e(01)=01110,e(10)=10101, e(11)=11011 is a group
code.
Decode the following words with maximum likelihood technique:
i)11110 ii)10011

(b) Find the particular solution of $a_r+5a_{r-1}+6a_{r-2} = 3r^2$ . **(10)**

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