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Code No. 3006/E

FACULTY OF SCIENCE

B.Sc. I-Semester (CBCS) Examination, November / December 2018

Subject : Mathematics

Paper – I : Differential Calculus

Time : 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

Note : Answer any FIVE of the following questions.

1 Expand y = log (sec x + tan x) is a Maclaurin series.

2 Verify Rolle's mean value theorem for the function
$f(x) = e^x (\sin x - \cos x)$ in $\left[\frac{p}{4}, \frac{5p}{4}\right]$

3 Evaluate $\lim_{x\to 0}(\cos x)^{\cot^2 x}$.

4 Find the radius of curvature of the curve
$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at any point θ.

5 If $u = \sin(xy^2)$, $x = \log t$, $y = e^t$ then find $\frac{du}{dt}$.

6 If $z = f(x - ay) + g(x + ay)$ then show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

7 Find the envelope of the curve
$x \cos^3\theta + y \sin^3\theta = C$, where C is any constant.

8 Find the asymptotes of the curve $r\theta = a$.

PART – B (4 x 15 = 60 Marks)
(Essay Answer Type)

Note: Answer ALL the questions.

9 (a) If $y = (x^2 - 1)^n$ then show that $(x^2 - 1)y_{n+2} + 2x\,y_{n+1} - n(n+1)y_n = 0$.

OR

(b) (i) State and prove Cauchy's mean value theorem.

(ii) If $f(x) = \sqrt{x}$, $g(x) = \frac{1}{\sqrt{x}}$ then find 'C' value of Cauchy's mean value theorem on [a, b].

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10 (a) Show that the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $(ax)^{2/3}-(by)^{2/3}=(a^2+b^2)^{2/3}$.

OR

(b) Find the circle of curvature at the point P(t^2, 2t) of the curve $y^2 = 4x$.

11 (a) (i) Expand f(x, y) = $x^2 - y^2 - 2x$ 4y + 1 as a Taylor series around P(1, 2).

(ii) If $x + y^2 - \sin xy = 1$ then evaluate $\frac{dy}{dx}$ at P(0, 1).

OR

(b) (i) If $u=\log\left(\frac{x^4+y^4}{x+y}\right)$ then evaluate $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.

(ii) If $f(x,y)=\begin{cases}\frac{x^2y(x-y)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ show that $f_{xy}\neq f_{yx}$ at O(0, 0)

12 (a) Find the minimum value $x^2 + y^2 + z^2$ when $xyz = a^3$.

OR

(b) Find the asymptotes of the curve
$y(x^2 - y^2) - 6y^2 + 5x - 7 = 0$

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