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FACULTY OF SCIENCE

B.Sc. III-Semester (CBCS) Examination, November / December 2018

Subject : Mathematics

Paper – III : Real Analysis (DSC)

Time : 3 Hours Max. Marks: 80

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)
Note : Answer any FIVE of the following questions.

1 Determine the limit of the sequence $\{s_n\}$, where $s_n = \sqrt{n^2+1} - n$.

2 Let $t_1=1$ and $t_{n+1} = \frac{t_n^2+2}{2t_n}$ for $n \geq 1$. Find the $\lim t_n$.

3 If $a_n = \sin\left(\frac{n\pi}{3}\right)$ then find lim sup a_n and lim inf a_n.

4 Show that $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges if and only if $p > 1$.

5 For $n = 0, 1, 2, 3, \ldots$, let $a_n = \left(\frac{4+2(-1)^n}{5}\right)^n$. Find lim sup $(a_n)^{\frac{1}{n}}$ lim inf $(a_n)^{\frac{1}{n}}$.

6 Let $f_n(x) = \frac{1+2\cos^2 nx}{\sqrt{n}}$. Prove that $\{f_n\}$ converges uniformly to 0 on **R**.

7 Prove that every continuous function f on [a, b] is integrable.

8 Show that $\left|\int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)\,dx\right| \leq \frac{16\pi^3}{3}$.

PART – B (4 x 15 = 60 Marks)
(Essay Answer Type)
Note: Answer ALL the questions.

9 (a) (i) If (s_n) converges to s, (t_n) converges to t, then prove that $(s_n t_n)$ converges to st.

(ii) If (s_n) converges to s and $s_n \neq 0$ for all n, and if $s \neq 0$, then show that $\left(\frac{1}{s_n}\right)$ converges to $\frac{1}{s}$.

OR

(b) (i) Prove that $\lim_{n\to\infty} a^n = 0$ if $|a| < 1$.

(ii) Prove that $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$.

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10 (a) (i) If the sequence (s_n) converges, then prove that every subsequence converges to the same limit.

(ii) State and prove Bolzano-Weierstrass theorem.

OR

(b) If (s_n) converges to a positive real number s and (t_n) is any sequence then prove that $\lim\sup s_n t_n = s \lim\sup t_n$.

11 (a) Let (f_n) be a sequence of functions defined and uniformly Cauchy on a set $S \subseteq R$. Then prove that there exists a function f on S such that $f_n \to f$ uniformly on S.

OR

(b) Derive an explicit formula for $\sum_{n=1}^{\infty} n^2 x^n$ for $|x| < 1$ and hence evaluate

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}.$$

12 (a) Let f be a bounded function on [a, b]. If P and Q are partitions of [a, b] and $P \subseteq R$, then prove that

$$L(f, P) \leq L(f, Q) \leq U(f, Q) \leq U(f, P).$$

OR

(b) Prove that a bounded function f on [a, b] in Riemann integrable on [a, b] $\Leftrightarrow$ it is Darboux integrable, in which case the values of the integrals agree.

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