120916474035

Code No. 3185/E

FACULTY OF SCIENCE

B.Sc. V-Semester (CBCS) Examination, November / December 2018

Subject : Mathematics

Paper – V : Linear Algebra (DSC)

Time : 3 Hours **Max. Marks: 60**

PART – A (5 x 3 = 15 Marks)
(Short Answer Type)

Note : Answer any FIVE of the following questions.

1 Let H be the set of all vectors of the form (a – 3b, b – a, a, b), where a and b are arbitrary scalars. Show that H is a subspace of $\mathbf{R}^4$.

2 Suppose $V_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}, V_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$ and $V_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$ are three vectors in $\mathbf{R}^3$. Then that that $\{v_1, v_2, v_3\}$ is a basis for $\mathbf{R}^3$.

3 Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}, c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$ and consider the bases for $\mathbf{R}^2$ given by $\beta = [b_1, b_2]$ and $\zeta \{c_1, c_2\}$. Find the change of coordinates matrix from β to ζ.

4 Find the eigne values of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$ and compare this result with eigen value of A^T.

5 Prove that an n x n matrix with n distinct eigen values in diagnalizable.

6 Define an inner product between two vectors and write the properties of inner product.

7 Given a matrix $A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$

Then rank of A and dir. Null A.

8 Let $b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\beta=[b_1, b_2]$ then find the coordinate vector $[x]_\beta$ of x relative to β.

PART – B (3 x 15 = 45 Marks)
(Essay Answer Type)
Note: Answer ALL the questions.

9 (a) (i) Prove that a set $\{v_1, v_2 \ldots v_p\}$ of two or more vectors with $v_1 \neq \bar{o}$, is linearly dependent if and only if some V_j (with $j > 1$) is a linear combination of the preceding vectors $v_1, v_2, \ldots v_{j-1}$.
(ii) Show that the set S={(1, 0, 0, -1) (0, 1, 0, -1) (0, 0, 1, -1) and (0, 0, 0, 1)} in R^4 is linearly independent.

OR

(b) (i) State and prove the spanning set theorem.
(ii) If a vector space V has a basis of n vectors then prove that every basis of V must consist of exactly n vectors.

10 (a) (i) Find the eigen values and eigen vectors a $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$.

(ii) Prove that the eigen values of a triangular matrix and its diagonal elements.

OR

(b) Define the term
(i) Rank of matrix (ii) Eigen values and Eigen vectors of matrix
(ii) Find the characteristic equation if

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

11 (a) Diagonalize the matrix, if possible

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

OR

(b) (i) Show that the set $\{u_1, u_2, u_3\}$ is an orthogonal set, where

$$u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \text{ and } u_3 = \begin{bmatrix} -1/2 \\ 2 \\ 7/2 \end{bmatrix}$$

(ii) If $S=\{u_1, u_2 \ldots u_p\}$ is an orthogonal set of non-zero vectors is R^n, then prove that S is linearly independent.

*****