Code No. 3120/E

FACULTY OF SCIENCE

B.Sc. IV-Semester (CBCS) Examination, May / June 2019

Subject : Mathematics

Paper – IV (SEC – 2) : (Number Theory)   Max. Marks: 40

Time : 1½ Hours

Note : Answer ALL the questions.

PART – A (2 x 5 = 10 Marks)
(Short Answer Type)

1 (a) Prove that there are infinity primes of the form 4n + 3 where n is a positive integer.

OR

(b) Find the remainder obtained when the sum $\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 + \dots \lfloor 100$ is divided by 12.

2 (a) Evaluate $\phi (36, 000)$.

OR

(b) Prove that for each positive integer $n \geq 1$, $\sum_{d/n} f(d) = n$.

PART – B (2 x 15 = 30 Marks)
(Essay Answer Type)

3 (a) (i) Show that $2^{20} - 1$ is divisible by 41.
   (ii) Prove that the functions τ and σ are both multiplicative functions.

OR

(b) (i) State and prove Mobius Inversion formula.
   (ii) If n is a positive integer and p is a prime, then prove that the exponent of the highest power of p that divides $\lfloor n$ is
   $$\sum_{k=1} \left[n / p^k \right]$$

4 (a) (i) If P is a prime and k > 0 then prove that $\phi(P^k) = P^k - P^{k-1}$.
   (ii) Prove that the function ϕ is a multiplicative function.

OR

(b) (i) State and Prove Euler's theorem.
   (ii) Show that $4^6 \equiv 1 \pmod 9$.

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