1209/3469050

Code No. 3121/E

FACULTY OF SCIENCE

B.Sc. IV-Semester (CBCS) Examination, May / June 2019

Subject : Mathematics
Paper – IV (DSC) : (Algebra)

Time : 3 Hours Max. Marks: 80

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

Note : Answer any FIVE of the following questions.

1 Prove that the set $GL(2, R) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} / a, b, c, d \in R, ad - bc \neq 0 \right\}$ Is a non abelian group with respect to matrix multiplication.

2 Let G be a group and H be a nonempty subset of G. If $ab \in H\ \forall a, b \in H$ and $a^{-1} \in H\ \forall\ a \in H$ then prove that H is a subgroup of G.

3 State and prove Lagrange's theorem.

4 A subgroup H of G is normal in G if and only if $xHx^{-1} \subseteq H\ \forall x \in H$.

5 Prove that the characteristic of an integral domain is either zero or prime.

6 Let R[x] denotes the set of all polynomials with real coefficients and let A denote the subset of all polynomials with constant term 0 then prove that A is an ideal of R [x] and A = < x >.

7 Let ϕ be a ring homomorphism from a ring R to a ring S then Ker $\varphi = \{r \in R / \varphi(r) = 0\}$ is an ideal of R.

8 If D is an integral domain then prove that D[x] is an integral domain.

PART – B (4 x 15 = 60 Marks)
(Essay Answer Type)

Note: Answer ALL from the questions.

9 (a) Every subgroup of a cyclic group is cyclic more over if $|\langle a \rangle| = n$ then the order of any subgroup of $\langle a \rangle$ is a divisor of n and for each positive divisor k of n, the group $\langle a \rangle$ has exactly one subgroup of order K namely $\langle a \rangle$.

OR

(b) Define Alternating group of degree n. Also prove that A_n has order $\frac{n!}{2}$ if n > 1.

10 (a) Prove that the group of rotations of a cube is isomorphic to S_4.

OR

(b) Let G be a group and let Z(G) be the centre of G. If $\frac{G}{Z(G)}$ is cyclic then G is abelian.

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11 (a) Prove that $Z_3[i] = \{a + ib \,/\, a, b \in Z_3\}$ is a field of order 9.

OR

(b) Let R be a commutative ring with unity and let A be an ideal of R then $\frac{R}{A}$ is an integral domain if and only if A is prime ideal.

12 (a) If R is a ring with unity and the characteristics of R is n > 0 then prove that R contains a subring isomorphic to Z_n. If the characteristic of R is 0 then R contains a subring isomorphic to Z.

OR

(b) Let $S = \left\{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \,/\, a, b \in R\right\}$ then show that $\phi : C \to S$ given by

$\phi(a + (b)) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is a ring isomorphism.

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