

Code No. 3318/E

FACULTY OF SCIENCE

B.Sc. VI-Semester (CBCS) Examination, May / June 2019

Subject : Mathematics

Paper – VIII (B) (DSE E-2) : (Vector Calculus)

Time : 3 Hours

Max. Marks: 60

PART – A (5 x 3 = 15 Marks)

(Short Answer Type)

Note : Answer any FIVE of the following questions.

1. If $\vec{F} = (y, x, z)$ and C is the curve given by $x = \cos \theta$, $y = \sin \theta$, $z = 0$, $(0 \leq \theta \leq 2\pi)$ then evaluate $\int_C \vec{F} \cdot d\vec{r}$.
2. Define conservative vector field with example.
3. Show that $\int_0^a \int_0^{b(1-x/a)} \int_0^{c(1-x/a-y/b)} dz dy dx = \frac{abc}{6}$.
4. Find the angle between the surfaces of the sphere $x^2 + y^2 + z^2 = 2$ and the cylinder $x^2 + y^2 = 1$ at a point where they intersect.
5. If $\vec{f} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$ find $\text{curl } \vec{f}$.
6. Find unit normal to the surface $y = x + z^2$ at the point $(1, 2, 1)$.
7. Give the physical interpretation of curl.
8. Show that $\text{curl} (\text{grad } \phi) = \vec{0}$ if ϕ is a scalar field.

PART – B (3 x 15 = 45 Marks)

(Essay Answer Type)

Note: Answer ALL from the questions.

9. (a) Evaluate the surface integral of $u = (y, x^2, z^2)$ over the surface S where S is the triangular surface on $x = 0$ with $y \geq 0$, $z \geq 0$, $y + z \leq 1$, with the normal \vec{n} directed in the positive direction of x -axis.
OR
(b) Evaluate the line integral $\int_C \vec{F} \times d\vec{r}$ where \vec{F} is the vector field $(y, x, 0)$ and C is the curve $y = \sin x$, $z = 0$ between $x = 0$ and $x = \pi$.
10. (a) A cube has a variable density given by $\rho = 1 + x + y + z$. What is the total mass of the cube?
OR
(b) Find the volume integral of the scalar field $\phi = x^2 + y^2 + z^2$ over the region V specified by $0 \leq x \leq 1$, $1 \leq y \leq 2$, $0 \leq z \leq 3$.
11. (a) Show that $\vec{u} = (y^2 z, -z^2 \sin y + 2xyz, 2z \cos y + y^2 x)$ is irrotational. Find the corresponding potential function.

OR

- (b) Find the gradient and Laplacian of $f = \sin(Kx) \sin(\ell y) \exp(\sqrt{K^2 + \ell^2} z)$.