Code No: 7114/E

1209-16-467-014

FACULTY OF SCIENCE

B.Sc. IV – Semester (CBSC) Examination, May / June 2018

Subject: Mathematics

Paper: IV Algebra

Time: 3 Hours **Max. Marks: 80**

SECTION – A (5 x 4 = 20 Marks)

(Short Answer Type)

Note: Answer any Five of the following questions

1. Let G be any group and $(ab)^2 = a^2 b^2$ for all a, b ∈ G then show that G is an abelian group.
2. If $\alpha, \beta \in S_5$ and $\alpha = (1\ 2\ 3\ 4\ 5)$, $\beta = (1\ 4\ 5\ 3\ 2)$ then evaluate $\alpha\beta$, $\alpha\beta^{-1}$, $\alpha^2\beta$.
3. If H and K are subgroups of a group G with $|H| = 24, |K| = 20$ then show that $H \cap K$ is an abelian group.
4. Determine all group homomorphisms from $\mathbf{Z}_{12}$ to $\mathbf{Z}_{30}$.
5. Define zero divisor in a ring R. Find all zero divisors in the ring $(\mathbf{Z}_{12}, +_{12}, \cdot_{12})$
6. If I_1 and I_2 are any two ideals in a ring R, then show that $I_1 + I_2 = \{ x + y / x \in I_1, y \in I_2\}$ is always an ideal of R.
7. Show that $f(x) = x^2 + 3x + 2$ has four zeros in $\mathbf{Z}_6$.
8. Let R, S be any two rings and $\phi : R \to S$ is a homo morphism. If R is communicative then show that ϕ(R) is commutative.

SECTION-B (4x15=60 Marks)

(Essay Answer Type)

9. (a) (i) Let G be a group and H is non empty subset of G. Then show that H is group of G if and only if $ab^{-1} \in H$ for all a,b ∈ H.
   (ii) In the symmetric group S_3 find the elements which satisfy $x^3 = e$ where e is the identity permutation of S_3.

   OR

   (b) (i) Show that every subgroup cyclic group is cyclic.
   (ii) Find all generators of the group $(\mathbf{Z}_8, +_8)$.

Contd…2….

10.(a) (i) State and prove Lagrange's theorem.
(ii) Show that every group of prime order is cyclic.

OR

(b) (i) Define center z(G) of a group G. Show that Z(G) is always a normal subgroup of G.
(ii) State and prove the first isomorphism theorem on groups.

11.(a) (i) Show that every finite integral domain is a field.
(ii) If a is an idempotent element in a ring R then show that 1-a is also idempotent.

OR

(b) (i) Define Maximal ideal in a ring R.
(ii) Let R be a cumulative ring with unity and A be an ideal of R then show That quotient $\frac{R}{A}$ is a field if and only if A is a maximal ideal.

12. (a) Let D be an integral domain. Then show that there exists a field F that contains a subring isomorphic to D.

OR

(b) Let F be a field and f(x), g(x) $\in$ F[x] with g(x) $\neq$ 0. Then show that there exists unique polynomials q(x) and r(x) in F[x] such that f(x) = q(x)g(x) + r(x) with either r(x) = 0 or deg r(x) <deg g(x).

****