

Code No. 7006 / E

FACULTY OF SCIENCE**B.Sc. I-Semester (CBCS) Examination, December 2017****Subject: Mathematics****Paper – I
Differential Calculus****Time: 3 Hours****Max.Marks: 80****PART – A (5x4 = 20 Marks)
[Short Answer Type]****Note: Answer any FIVE of the following questions.**

- 1 Find the n^{th} derivative of $f(x) = \frac{1}{6x^2 - 5x + 1}$.
- 2 Expand $f(x) = e^x$ in powers of $(x-2)$.
- 3 Evaluate $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$.
- 4 Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $P(1,1)$.
- 5 If $w = x^2 + y^2$, $x = r-s$ and $y = r+s$ then evaluate $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$.
- 6 If $z = f(x+ay) + g(x-ay)$ then show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.
- 7 Find the envelope of the family of circles $x^2 + y^2 - 2ax \cos \alpha - 2ay \sin \alpha = c^2$ where α is the parameter.
- 8 Find the asymptotes of the curve $\gamma = \frac{a\theta}{\theta-1}$.

**PART – B (4x15 = 60 Marks)
[Essay Answer Type]****Note: Answer ALL the questions.**

- 9 a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$.

OR

- b) i) State and prove Rolle's mean value theorem.
ii) If Rolle's mean value theorem holds for the function $f(x) = x^3 + ax^2 + bx$, $1 \leq x \leq 2$ at the point $x = \frac{4}{3}$ then find the values of a and b .

10 a) Find the circle of curvature of the curve $x = a (\cos t + t \sin t)$, $y = a (\sin t - t \cos t)$ at

$$t = \frac{\pi}{4}.$$

OR

b) i) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right).$

ii) Evaluate $\lim_{x \rightarrow \frac{\pi}{4}} (\tan x)^{\tan 2x}.$

11 a) i) If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0.$

ii) If $x^y + y^x = a^b$ then find $\left(\frac{dy}{dx} \right).$

OR

b) Expand $f(x,y) = e^x \cos y$ in terms of $(x-1)$ and $\left(y - \frac{\pi}{4} \right)$ using Taylor's theorem.

12 a) Find the asymptotes of the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0.$

OR

b) Find the minimum value of $x + y + z$, subject to the condition $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1.$
