B.Tech. Fourth Semester (Chem. / Poly / Food / Pulp / Oil / Petro) (Old)

Applied Mathematics – III : 4 SCE 1

P. Pages : 3 **AW - 3554**

Time : Three Hours Max. Marks : 80

Notes :
1. Answer **three** question from Section A and **three** question from Section B.
2. Due credit will be given to neatness and adequate dimensions.
3. Assume suitable data wherever necessary.
4. Illustrate your answer necessary with the help of neat sketches.
5. Use of calculator is permitted.
6. Use of pen Blue/Black ink/refill only for writing the answer book.

SECTION – A

1. a) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x^2\cos x$ **6**

b) Solve $(D^2+3D+2)y=e^{e^x}$ by using variation of parameter. **7**

OR

2. a) Solve $(D^3+1)y=\sin 3x-\cos^2\frac{x}{2}$ **6**

b) Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=\cos(\log x)+x\sin(\log x)$ **7**

3. a) Evaluate Laplace transform of $\int_0^t e^t\frac{\sin t}{t}dt$ **5**

b) Prove that $L^{-1}\left\{\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)\right\}=\int_0^t\frac{2(1-\cos u)}{u}du$ **4**

c) Express f(t) in terms unit step function and hence find its Laplace transform **5**

$$f(t)=t^2\ ,\ 0<t<1$$
$$=4t\ ,\ t>1$$

OR

4. a) Find the Laplace transform of **7**

$$f(t)=a\sin pt\ ,\quad 0<t<\frac{\pi}{p}$$
$$=0\ ,\quad \frac{\pi}{p}<t<\frac{2\pi}{p}$$

where $f\left(t+\frac{2\pi}{p}\right)=f(t)$

b) Use convolution theorem to find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$ 7

5. a) Solve the differential equation using Laplace transform 7

$$\frac{d^2y}{dt^2}+9y=\cos 2t \text{ if } y(0)=1,\ y\left(\frac{\pi}{2}\right)=-1$$

b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ 6

OR

6. a) Using Fourier integral show that 7

$$\int_0^{\infty}\frac{\sin\pi\lambda\sin x\lambda}{1-\lambda^2}d\lambda=\frac{\pi}{2}\sin x \quad , \quad 0<x<\pi$$
$$=0 \quad , \quad x>\pi$$

b) Use Laplace transform to solve the differential equation $\frac{dy}{dt}+2y+\int_0^t y\,dt=\sin t$ when y (0) = 1. 6

SECTION - B

7. a) Solve the following difference equation -

i) $y_{x+2}+y_{x+1}+y_x=x^2$ 4

ii) $y_{n+2}-4y_n=n^2+n-1$ 4

b) Find the inverse z-transform of $\frac{z}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}$ 6

OR

8. a) Solve the difference equation $y_{n+2}-2\cos\alpha\, y_{n+1}+y_n=0$ with y(0) = 1, y(1) = $\cos\alpha$ using method of z-transform. 6

b) Find the z-transform of

i) $\frac{1}{n+1}$ 4

ii) $(\cos\theta+\sin\theta)^n$ 4

9. a) Find the tangential & normal component of acceleration at any time t of a particle whose position (x, y) at any time t is given by $x=\log(t^2+1),\ y=t-2\tan^{-1}t$. 6

b) Find the directional derivative of $\phi = e^{2x} \cos yz$ at the origin in the direction of the tangent to the curve $x = a\sin t,\ y = a\cos t,\ z = at$ at $t = \frac{\pi}{4}$ 7

OR

10. a) If $\rho\overline{F} = \nabla P$, where ρ, P and $\overline{F}$ are point functions, prove that $\overline{F}\cdot \text{curl}\,\overline{F} = 0$. 6

b) Prove that $\overline{a}\cdot\nabla\left(\overline{b}\cdot\nabla\frac{1}{r}\right) = \frac{3(\overline{a}\cdot\overline{r})\,(\overline{b}\cdot\overline{r})}{r^5} - \frac{(\overline{a}\cdot\overline{b})}{r^3}$ 7

11. a) A vector field is given by $\overline{F} = \sin y\,\hat{i} + x(1+\cos y)\,\hat{j}$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2,\ z = 0$. 6

b) Apply Stokes theorem to evaluate $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$ where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0), (0, 0, 6). 7

OR

12. a) Use Divergence theorem to evaluate $\iint_s \left(y^2z^2\hat{i} + z^2x^2\hat{j} + x^2y^2\hat{k}\right)\cdot d\overline{s}$ where s is the upper part of the sphere $x^2 + y^2 + z^2 = 1$. 6

b) Prove that $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (x^2 - yz)\hat{k}$ is irrotational and find ϕ if $\overline{F} = \nabla\phi$. 7

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