B.Tech. Fourth Semester (Chem. / Poly / Food / Pulp & Paper / Oil & Paint / Petro) (Old)

Applied Mathematics – II : 4 SCT 1

P. Pages : 2 | AW - 3553

Time : Three Hours | Max. Marks :

* 0 2 9 5 *

Notes : 1. Assume suitable data wherever necessary.
2. Use of calculator normal distribution table significance table is permitted.
3. Use of pen Blue/Black ink/refill only for writing the answer book.

1. A tightly stretched string with fixed end point x = 0 and x = L, is initially in a position given by $y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{L}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end and at any time t. **13**

OR

2. A rod of length L has its ends A and B kept at 0°C and 100°C respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained. Find the temperature u(x, t) at a distance x from A and at any time t. **13**

3. a) Show that the function $u = x^3 - 3xy^2$ is harmonic. Find its harmonic conjugate function and corresponding analytic function in terms of z. **7**

b) Prove that $\cosh^2 z - \sinh^2 z = 1$. **3**

c) Find all values of z, which satisfies the equation $z^4 + 1 = 0$. **4**

OR

4. a) If F(z) is regular function, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|F'(z)|^2$ **6**

b) Prove that $(x+iy)^{m/n} + (x-iy)^{m/n} = 2\left(\sqrt{x^2+y^2}\right)^{m/n} \cdot \cos\left(\frac{m}{n}\tan^{-1}(y/x)\right)$ **5**

c) Prove that $\log(1 + i\tan\alpha) = \log\sec\alpha + i\alpha$. **3**

5. a) Find the root of the equation $x^3 + x - 1 = 0$ by iterative method. **6**

b) Evaluate $\int_0^1 x^3\,dx$ by using Trapezoidal rule, with five sub intervals. **7**

OR

6. a) If $y(1) = -3$, $y(3) = 9$, $y(4) = 30$, $y(6) = 132$; find the Lagrange's interpolation polynomial that takes the same values as y at the given point. **6**

b) Given that **7**

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.99	8.40	8.78	9.13	9.45	9.75	10.03

Find $\frac{dy}{dx}$ at x = 1.1.

7. a) Solve the LPP graphically - 6

Maximize $Z = 2x_1 + x_2$

subject to $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

with $x_1 \geq 0, x_2 \geq 0$

b) Solve the LPP by using simplex method : 7

Maximize $Z = 3x_1 + 4x_2$

subject to $2x_1 + 3x_2 \leq 9$

$4x_1 + 3x_2 \leq 12$

with $x_1 \geq 0, x_2 \geq 0$

OR

8. a) Prove that "the feasible region of LPP is convex". 6

b) Solve the following LPP using simplex method and comment on the solution. 7

Minimize $f = -3x_2 - 2x_2$

subject to $x_1 - x_2 \leq 1,$

$3x_1 - 2x_2 \leq 6$

with $x_1 \geq 0, x_2 \geq 0$

9. a) For the following distribution, compute the mean and S.D. of 100 students. 7

Mass in kg	60 - 62	63 - 65	66 - 68	69 - 71	72 - 74
No. of students	05	18	42	27	08

b) The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively; find $P(x \geq 1)$. 6

OR

10. a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals - 6

i) exactly 3 ii) at least 2 iii) none will suffer a bad reaction.

b) Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and S.D. of 5. What is the percentage of students scored more than 60 marks ? 7

11. a) Using samples of sizes 10 and 16 with variances $S_1^2 = 50$ and $S_2^2 = 30$; assuming normality of the population, test the hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative $\sigma_1^2 > \sigma_2^2$. Choose $\alpha = 5\%$. 7

b) Find the regression line of y on x for the data. 7

x	1	2	3	4	5
y	3	2	5	1	4

OR

12. a) Distinguish between - 4+4

i) Null and Alternative hypothesis. ii) Type - I and Type - II errors.

b) In a sample of 600 men from a certain city, 450 are found smokers. In another sample of 900 men from another city 450 are smokers. Do the data indicate that the cities are significantly different with respect to the habit of smoking among men ? 6

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