B.Sc. (Part—III) Semester—V Examination

5S : MATHEMATICS (New)

(Mathematical Methods)

Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No.1 is compulsory and attempt it once.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternative (1 mark each) :

(i) If $p_n(x)$ is the solution of Legendre's D.E., then $p_n(-1)$ is :

(a) -1 (b) 1

(c) $(-1)^n$ (d) 0

(ii) The value of integral $\int_{-1}^{1} x^2 p_1(x)\, dx$, where $p_1(x)$ is Legendre's polynomial of degree 1, equals :

(a) $\frac{2}{3}$ (b) $\frac{4}{35}$

(c) $\frac{4}{15}$ (d) 0

(iii) The value of $J_{1/2}(x)$ equals :

(a) $\sqrt{\frac{2}{n\pi}} \cos x$ (b) $\sqrt{\frac{2}{n\pi}} \sin x$

(c) $\sqrt{\frac{n\pi}{2}} \cos x$ (d) $\sqrt{\frac{n\pi}{2}} \sin x$

(iv) Eigen functions corresponding to different Eigen values are :

(a) Linearly dependent (b) Linearly independent

(c) Real (d) None

(v) The coefficient in a half range sine series for the function $f(x) = \sin x$ defined on $[0, \ell]$ is given by :

(a) $\int_0^{\ell} \sin x \cos \frac{n\pi x}{\ell} dx$ (b) $\int_0^{\ell} \cos x \cos \frac{n\pi x}{\ell} dx$

(c) $\frac{2}{\ell}\int_0^{\ell} \sin x \sin \frac{n\pi x}{\ell} dx$ (d) $\frac{2}{\ell}\int_0^{\ell} \sin x \sin \frac{n\pi x}{\ell} dx$

(vi) The function $f(x) = (-\sin x)$ is :

(a) Odd (b) Even

(c) Even and Odd (d) None of these

(vii) If $L[f(t)] = F(s)$, then $L[f(at)]$ is :

(a) $F(s-a)$ (b) $\frac{1}{a}F\left(\frac{s}{a}\right)$

(c) $F\left(\frac{s}{a}\right)$ (d) $aF\left(\frac{s}{a}\right)$

(viii) The value of $L^{-1}\left[\frac{1}{s-a}\right]$ is :

(a) 1 (b) t

(c) e^t (d) e^{at}

(ix) The Fourier sine transform of $f(x) = e^{|-x|}$, $x \geq 0$ is :

(a) $\frac{\lambda}{1+\lambda^2}$ (b) $\frac{\lambda}{1-\lambda^2}$

(c) $\frac{2\lambda}{1-\lambda^2}$ (d) $\frac{1}{1+\lambda^2}$

(x) If $F[f(x)] = F(\lambda)$, then the Fourier transform of $f(ax)$ is :

(a) $F\left(\frac{\lambda}{a}\right)$ (b) $\frac{1}{|a|}F\left(\frac{\lambda}{a}\right)$, $a = 0$

(c) $\frac{1}{|a|}F(\lambda)$ $a \neq 0$ (d) $\frac{1}{|a|}F\left(\frac{\lambda}{a}\right)$ $a \neq 0$ 10

UNIT—I

2. (a) Show that $p_n(x)$ is the coefficient of h^n in the ascending power series expansion of $(1 - 2xh + h^2)^{-1/2}$. 5

(b) Prove that $np_n = xp_n^1 - p_{n-1}^1$. 3

(c) Prove that $x^2 = \frac{1}{3}p_0(x) + \frac{2}{3}p_2(x)$. 2

3\. (p) Prove that $\int_{-1}^{1}[p_x(x)]^2 dx = \frac{2}{2n+1}$. 5

(q) Prove that $p_x(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. 5

UNIT—II

4\. (a) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$. 4

(b) Prove that $xJ_p^1 = pJ_p - xJ_{p+1}$. 4

(c) Evaluate $\int_a^b J_0(x)\cdot J_1(x)\,dx$. 2

5\. (p) Prove that Eigen values of the S-L problem are real. 4

(q) Prove that $\left(x^p \cdot J_p\right)' = x^p J_{p-1}$ 3

(r) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. 3

UNIT—III

6\. (a) If the trigonometric series $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ converges uniformly to f(x) in $c \le x < c + 2\pi$, then find the Fourier coefficient of f(x). 5

(b) Obtain Fourier Series in [0, 2] for the function $f(x) = x^2$. 5

7\. (p) Obtain Fourier Series in $[-\pi, \pi]$ for the function :

$$f(x) = \begin{cases} -\pi & , \ -\pi < x < 0 \\ x & , \ 0 < x < \pi \end{cases}$$ 5

(q) Obtain Fourier cosine series in $[0, \pi]$ for the function $f(x) = \sin x$. 5

UNIT—IV

8\. (a) Prove that $L[t^n \cdot f(t)] = (-1)^n \frac{d^n}{ds^n}F(s)$, n = 1,2,3..... 4

(b) Find L[sin t · cos 2t · cos 3t]. 3

(c) Show that $L(t^n) = \frac{n!}{s^{n+1}}$, $s > 0$. 3

9. (p) Solve the D.E. $y'' + 4y' = -8t,\ y(0) = y'(0) = 0.$ 4

(q) Find the inverse Laplace transform of $\dfrac{1}{(s-2)(s+2)^2}$ by using Convolution theorem. 3

(r) Prove that $L(u_{tt}) = s^2L(u(x, t)) - su(x, 0) - u_t(x, 0).$ 3

UNIT—V

10. (a) Find the finite Fourier sine and cosine transform of $f(x) = \sin ax$ in $(0, \pi)$. 4

(b) Find the Fourier transform of the function :

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$ 4

(c) Prove that $\int_0^{\ell} f'(x) \sin\frac{n\pi x}{\ell}\,dx = -\frac{n\pi}{\ell}F_c(n)$. 2

11. (p) Find the Fourier sine and cosine transform of the function $f(x) = x^{n-1}$, $n > 0$. 5

(q) Find finite Fourier cosine transform of u_x and u_{xx}; where $u = u(x, t)$. 5