

B.Sc. Part-I (Semester-I) Examination
MATHEMATICS

(Differential & Integral Calculus)

Paper—II

Time : Three Hours]

[Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives (1 mark each) :

10

(i) The value of $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ is :

(a) 0

(b) 1

(c) ∞

(d) None of these

(ii) If $y = e^{-2x}$, then y_{11} is :

(a) $-2^{11} e^{-2x}$

(b) $2^{11} e^{-2x}$

(c) $-2^{11} e^{2x}$

(d) None of these

(iii) The series :

$$x - \frac{x^3}{3!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + \dots$$

is the expansion of function :

(a) $\sin x$

(b) $\sinh x$

(c) $\cos x$

(d) $\cosh x$

(iv) $|x - x_0| < \delta$ represents :

(a) $x_0 - \delta < x < x_0 + \delta$

(b) $x_0 + \delta < x < x_0 - \delta$

(c) $x_0 - \delta \leq x < x_0 + \delta$

(d) $x_0 - \delta < x \leq x_0 + \delta$

(v) If f be differentiable on (a, b) and $f'(x) = 0, \forall x \in [a, b]$, then $f(x)$ is :

(a) Monotonic increasing in $[a, b]$

(b) Monotonic decreasing in $[a, b]$

(c) Constant in $[a, b]$

(d) None of these

(vi) For $f(x) = x^2$; in $[1, 3]$ then the value of 'C' by Lagrange's mean value theorem is :

(a) $\frac{6}{13}$

(b) 2

(c) 0

(d) 1

(vii) The area bounded by the curve $x = g(y)$, y -axis and $y = a$, $y = b$ is :

(a) $\int_a^b y \, dx$

(b) $\int_a^b x \, dy$

(c) $\int_a^b y^2 \, dx$

(d) $\int_a^b x^2 \, dy$

(viii) The functions f and g be :

(i) continuous in $[a, b]$

(ii) derivable in (a, b) and

(iii) $g'(x) \neq 0$ for all $x \in (a, b)$.

These are the hypothesis of mean value theorem by :

(a) Rolle's

(b) Lagrange's

(c) Cauchy's

(d) Leibnitz

(ix) The function $f(x)$ has the removable discontinuity if :

(a) $f(x^+) \neq f(x^-)$

(b) $f(x^+) = f(x^-) \neq f(x)$

(c) $f(x^+)$, $f(x^-)$ do not exist

(d) None of these

(x) $\frac{d}{dx} \cosh x$ is :

(a) $\sinh x$

(b) $-\sinh x$

(c) $h \sinh x$

(d) $-h \sinh x$

UNIT—I

2. (a) If $\lim_{x \rightarrow x_0} f(x) = \ell$ and $\lim_{x \rightarrow x_0} g(x) = m$, then prove that :

$$\begin{aligned} \lim_{x \rightarrow x_0} [f(x) + g(x)] &= \lim_{x \rightarrow x_0} f(x) + \lim_{x \rightarrow x_0} g(x) \\ &= \ell + m. \end{aligned}$$

4

(b) Prove that the function defined by $f(x) = x^2$ is continuous for all $x \in \mathbb{R}$.

3

(c) Using definition of limit, prove that :

$$\lim_{x \rightarrow 3} \frac{x^3 - 2x^2 - x - 6}{x - 3} = 14.$$

3

3. (p) Define limit of a function and show that the limit of a function if it exist, is unique.

1+3

(q) Prove that $\lim_{x \rightarrow 2} x^2 = 4$; by using ϵ - δ definition.

3

(r) If $f(x) = \frac{e^{1/x}}{1 + e^{1/x}}$, $x \neq 0$, www.FirstRanker.com

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$$= 0, \quad x = 0$$

then show that $f(x)$ has a simple discontinuity at $x = 0$.

3

UNIT—II

4. (a) Prove that if $f(x)$ is differentiable at $x = x_0$, then it is continuous at $x = x_0$. Is converse of this statement true ? Justify. 5

(b) Evaluate :

$$\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}$$

3

- (c) If $y = A \sin mx + B \cos mx$, then prove that $y_2 + m^2 y = 0$. 2

5. (p) If $y = \sin(m \sin^{-1} x)$, then show that :

$$(i) (1 - x^2)y_2 - xy_1 + m^2 y = 0$$

$$(ii) (1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

5

- (q) If $y = \frac{1}{ax + b}$, then prove that $y_n = \frac{(-1)^n n! a^n}{(ax + b)^{n+1}}$. 3

(r) Evaluate :

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$$

2

UNIT—III

6. (a) State and prove Lagrange's mean value theorem. 4

(b) Verify Cauchy mean value theorem for the functions :

$$f(x) = e^x \text{ and } g(x) = e^{-x} \text{ in } [a, b].$$

3

- (c) Expand $\sin x$ in powers of $x - \frac{\pi}{2}$, upto first four terms. 3

7. (p) State and prove Cauchy's mean value theorem. 4

(q) Expand $3x^3 + 4x^2 + 5x - 3$ about the point $x = 1$ by Taylor's theorem. 3

(r) Verify the Rolle's theorem for the function :

$$f(x) = \frac{\sin x}{e^x} \text{ in } [0, \pi].$$

3

UNIT—IV

8. (a) If $u = f(x, y, z)$ is a homogeneous function of degree n , then show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = nu.$$

4

(c) If $u = e^x (x \cos y - y \sin y)$, then find the value $u_{xx} + u_{yy}$. 3

9. (p) If $u = f(x, y)$ be homogeneous function of degree n then prove that :

(i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ are homogeneous functions of degree ' $n - 1$ ' in x, y and

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$. 4

(q) If $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$, then show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0. \quad 3$$

(r) If $u = \log \frac{x^4 - y^4}{x - y}$, $x \neq y$, then prove that :

(i) $x u_x + y u_y = 3$

(ii) $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = -3$. 3

UNIT—V

10. (a) Prove that :

$$\int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x \, dx. \quad 4$$

(b) Evaluate :

$$\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + x + 1}} \, dx. \quad 3$$

(c) Show that ' $8a$ ' is the length of an arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$; $0 \leq t \leq 2\pi$. 3

11. (p) Prove that :

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - I_{n-2}.$$

Hence evaluate $\int \tan^3 x \, dx$. 4

(q) Find the area bounded by the x -axis, the curve $y = c \cosh \frac{x}{c}$ and the ordinates $x = 0$, $x = a$. 3

(r) Show that length of the curve $y = \log \sec x$ between the points, where $x = 0$ and $x = \frac{\pi}{3}$ is $\log_e (2 + \sqrt{3})$. 3