

B.Sc. Part-I (Semester-I) Examination**MATHEMATICS****(Differential & Integral Calculus)****Paper-II**

Time : Three Hours]

[Maximum Marks : 60]

Note :— (1) Question No. 1 is compulsory. Attempt once.(2) Attempt **ONE** question from each unit.**1.** Choose the correct alternatives (1 mark each) :

10

(i) The value of $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ is :

(a) 0

(b) 1

(c) ∞

(d) None of these

(ii) If $y = e^{-2x}$, then y_{11} is :(a) $-2^{11} e^{-2x}$ (b) $2^{11} e^{-2x}$ (c) $-2^{11} e^{2x}$

(d) None of these

(iii) The series :

$$x - \frac{x^3}{3!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + \dots$$

is the expansion of function :

(a) $\sin x$ (b) $\sinh x$ (c) $\cos x$ (d) $\cosh x$ (iv) $|x - x_0| < \delta$ represents :(a) $x_0 - \delta < x < x_0 + \delta$ (b) $x_0 + \delta < x < x_0 - \delta$ (c) $x_0 - \delta \leq x < x_0 + \delta$ (d) $x_0 - \delta < x \leq x_0 + \delta$ (v) If f be differentiable on (a, b) and $f'(x) = 0, \forall x \in [a, b]$, then $f(x)$ is :(a) Monotonic increasing in $[a, b]$ (b) Monotonic decreasing in $[a, b]$ (c) Constant in $[a, b]$

(d) None of these

(vi) For $f(x) = x^2$; in $[1, 3]$ then the value of 'C' by Lagrange's mean value theorem is :(a) $\frac{6}{13}$

(b) 2

(c) 0

(d) 1

(a) $\int_a^b y \, dx$

(b) $\int_a^b x \, dy$

(c) $\int_a^b y^2 \, dx$

(d) $\int_a^b x^2 \, dy$

(viii) The functions f and g be :

- (i) continuous in $[a, b]$
- (ii) derivable in (a, b) and
- (iii) $g'(x) \neq 0$ for all $x \in (a, b)$.

These are the hypothesis of mean value theorem by :

- | | |
|--------------|----------------|
| (a) Rolle's | (b) Lagrange's |
| (c) Cauchy's | (d) Leibnitz |

(ix) The function $f(x)$ has the removable discontinuity if :

- | | |
|-----------------------------------|---------------------------------|
| (a) $f(x^+) \neq f(x^-)$ | (b) $f(x^+) = f(x^-) \neq f(x)$ |
| (c) $f(x^+), f(x^-)$ do not exist | (d) None of these |

(x) $\frac{d}{dx} \cosh x$ is :

- | | |
|-----------------|------------------|
| (a) $\sinh x$ | (b) $-\sinh x$ |
| (c) $h \sinh x$ | (d) $-h \sinh x$ |

UNIT—I

2. (a) If $\lim_{x \rightarrow x_0} f(x) = \ell$ and $\lim_{x \rightarrow x_0} g(x) = m$, then prove that :

$$\lim_{x \rightarrow x_0} [f(x) + g(x)] = \lim_{x \rightarrow x_0} f(x) + \lim_{x \rightarrow x_0} g(x)$$

$$= \ell + m.$$

4

(b) Prove that the function defined by $f(x) = x^2$ is continuous for all $x \in \mathbb{R}$.

3

(c) Using definition of limit, prove that :

$$\lim_{x \rightarrow 3} \frac{x^3 - 2x^2 - x - 6}{x - 3} = 14.$$

3

3. (p) Define limit of a function and show that the limit of a function if it exist, is unique.

1+3

(q) Prove that $\lim_{x \rightarrow 2} x^2 = 4$; by using ϵ - δ definition.

3

$$= 0, \quad , x = 0$$

then show that $f(x)$ has a simple discontinuity at $x = 0$.

3

UNIT-II

4. (a) Prove that if $f(x)$ is differentiable at $x = x_0$, then it is continuous at $x = x_0$. Is converse of this statement true ? Justify.

5

- (b) Evaluate :

$$\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}.$$

3

- (c) If $y = A \sin mx + B \cos mx$, then prove that $y_2 + m^2y = 0$.

2

5. (p) If $y = \sin(m \sin^{-1} x)$, then show that :

$$(i) (1 - x^2)y_2 - xy_1 + m^2y = 0$$

$$(ii) (1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

5

- (q) If $y = \frac{1}{ax + b}$, then prove that $y_n = \frac{(-1)^n n! a^n}{(ax + b)^{n+1}}$.

3

- (r) Evaluate :

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}.$$

2

UNIT-III

6. (a) State and prove Lagrange's mean value theorem.

4

- (b) Verify Cauchy mean value theorem for the functions :

$$f(x) = e^x \text{ and } g(x) = e^{-x} \text{ in } [a, b].$$

3

- (c) Expand $\sin x$ in powers of $x - \frac{\pi}{2}$, upto first four terms.

3

7. (p) State and prove Cauchy's mean value theorem.

4

- (q) Expand $3x^3 + 4x^2 + 5x - 3$ about the point $x = 1$ by Taylor's theorem.

3

- (r) Verify the Rolle's theorem for the function :

$$f(x) = \frac{\sin x}{e^x} \text{ in } [0, \pi].$$

3

UNIT-IV

8. (a) If $u = f(x, y, z)$ is a homogeneous function of degree n , then show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = nu.$$

4

- (c) If $u = e^x (x \cos y - y \sin y)$, then find the value $u_{xx} + u_{yy}$. 3
9. (p) If $u = f(x, y)$ be homogeneous function of degree n then prove that :

(i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ are homogeneous functions of degree ' $n - 1$ ' in x, y and

$$(ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u. \quad 4$$

- (q) If $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$, then show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0. \quad 3$$

- (r) If $u = \log \frac{x^4 - y^4}{x - y}$, $x \neq y$, then prove that :

(i) $x u_x + y u_y = 3$

(ii) $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = -3.$ 3

UNIT—V

10. (a) Prove that :

$$\int \sin^m x \cos^n x dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x dx. \quad 4$$

- (b) Evaluate :

$$\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + x + 1}} dx. \quad 3$$

- (c) Show that '8a' is the length of an arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$; $0 \leq t \leq 2\pi.$ 3

11. (p) Prove that :

$$\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - I_{n-2}.$$

Hence evaluate $\int \tan^3 x dx.$ 4

- (q) Find the area bounded by the x-axis, the curve $y = c \cosh \frac{x}{c}$ and the ordinates $x = 0$, $x = a.$ 3

- (r) Show that length of the curve $y = \log \sec x$ between the points, where $x = 0$ and $x = \frac{\pi}{3}$ is $\log_e (2 + \sqrt{3}).$ 3