

B.Sc. Part-I (Semester-II) Examination
MATHEMATICS
(Differential Equations : Ordinary & Partial)
Paper—III

Time : Three Hours]

[Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory. Solve it in **ONE** attempt only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

- (i) The roots of the equation $(D^2 - 4D + 13)^2 y = 0$ are : 1
 (a) distinct and real (b) real and equal
 (c) complex and repeated (d) None of these
- (ii) A linear equation of first order is of the form $Y' + PY = Q$ in which ? 1
 (a) P is function of Y
 (b) P and Q are function of X
 (c) P is function of X and Q is function of Y
 (d) None of these
- (iii) The condition for the partial differential equation $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ to be compatible is that : 1
 (a) $J_{pp} + J_{yq} + PJ_{zp} + qJ_{zq} = 0$ (b) $J_{xp} + J_{yq} + PJ_{zp} + qJ_{zq} = 0$
 (c) $J_{xp} + J_{yq} + PJ_{zp} + qJ_{zq} = 0$ (d) None of these
- (iv) The D.E. $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 v}{\partial t^2} = 0$ is called : 1
 (a) Partial differential equation (b) Ordinary differential equation
 (c) Total differential equation (d) Linear differential equation
- (v) An equation of the form $Pp + Qq = R$ where P, Q, R are the functions of X, Y, Z is called : 1
 (a) Lagrange's equation (b) Jacobi's equation
 (c) Charpit's equation (d) Clairaut's equation
- (vi) The particular solution of DE $W'' + PW' + QW = 0$ is $y = e^x$ iff : 1
 (a) $P + xQ = 0$ (b) $1 + p + q = 0$
 (c) $1 - P + Q = 0$ (d) $m^2 + mP + Q = 0$
- (vii) The solution of PDE $(D - mD')z = 0$ is : 1
 (a) $z = F(y + mx)$ (b) $z = F'(y - mx)$
 (c) $z = F(e^{xy})$ (d) None of these

(a) $F(x, y, z, p) = 0$

(b) $F(x, y, z, q) = 0$

(c) $F(x, y, z, p, q) = 0$

(d) $F(y, z, p, q) = 0$

 (ix) The complete integral of $F(x, p) = G(y, q)$ is :

1

(a) $z = \int h(x) dx$

(b) $\int k(y) dy$

(c) $z = \int h(x) dx + \int k(y) dy + b$

(d) None of these

 (x) The DE $Mdx + Ndy = 0$ is exact iff :

1

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

(d) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

UNIT—I

2. (a) Show that the D.E. :

$$(\sin x \sin y - x e^y) dy = (e^y + \cos x \cdot \cos y) dx$$

is exact and hence solve it.

5

 (b) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$.

5

3. (p) Solve the D.E. :

$$(1 + x^2) dy + 2xy dx = \cot x dx.$$

5

(q) Solve :

$$xy - \frac{dy}{dx} = y^3 e^{-x^2}.$$

5

UNIT—II

 4. (a) Solve the D.E. $(D^2 - 4)y = e^{2x}$.

5

 (b) Solve the D.E. $(x^2 D^2 - 3xD + 5)y = x^2 \sin(\log x)$.

5

 5. (p) Solve the D.E. $(x^2 D^2 - xD + 4)y = \cos(\log x)$.

5

 (q) Solve the D.E. $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = e^{2x} + \sin 2x$.

5

UNIT—III

 6. (a) Solve the system of D.E. : $D^3 x - 2y = 0$ and $D^2 y + 2x = 0$.

5

 (b) Solve the D.E. $y'' - y = \frac{2}{1 + e^x}$ by variation of parameter.

5

7. (p) Solve $x^2y'' + xy' + 10y = 0$ by changing the independent variable from x to $z = \log x$. 5

- (q) Solve the following D.E. by removing the first derivative :

$$x \frac{d}{dx} \left(x \frac{dy}{dx} - y \right) - 2x \frac{dy}{dx} + 2y + x^2y = 0. \quad 5$$

UNIT—IV

8. (a) Solve :

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}. \quad 5$$

- (b) Find the complete integral of $z = p^2x + q^2y$. 5

9. (p) Find the general solution of PDE $x^2p + y^2q = (x + y)z$. 5

- (q) Solve the PDE $p^2 + q^2 = k^2$. 5

UNIT—V

10. (a) Solve the D.E. $(D^2 + 3DD' + 2D'^2)z = x + y$. 5

- (b) Solve by Charpits method $pxy + pq + qy = yz$. 5

11. (p) The PDE $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ where f is homogeneous in x, y, z . Prove this. 5

- (q) Find a real function v of x and y , reducing to zero when $y = 0$ and satisfying

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -4\pi(x^2 + y^2). \quad 5$$

