B. Sc. (Part–I) Semester—II Examination

MATHEMATICS

(Vector Analysis and Solid Geometry)

Paper–IV

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory; attempt it once only.

(2) Attempt one question from each unit.

1. Choose the correct alternative :

(i) If three vectors $\bar{a}$, $\bar{b}$, $\bar{c}$ are coplanar, then for scalar triple product, which of the following is correct ?

(a) $\bar{b} \times \bar{c}$ is perpendicular to the vector $\bar{a}$

(b) $\bar{b} \times \bar{c}$ is parallel to the vector $\bar{a}$

(c) $\bar{b} \times \bar{c}$ is equal to the vector $\bar{a}$

(d) None of these. 1

(ii) The scalar triple product represents the volume of the _____.

(a) rectangle (b) sphere

(c) parallelepiped (d) ellipse 1

(iii) The curvature k is determined _______.

(a) only in magnitude (b) only in sign

(c) both in magnitude and sign (d) neither in magnitude nor sign 1

(iv) A plane determined by the tangent and binormal at $P(\vec{r})$ to the curve $\vec{r} = \vec{r}(s)$ is a _______.

(a) osculating plane (b) rectifying plane

(c) normal plane (d) none of these 1

(v) Which of the following quantity is defined ?

(a) div $(\text{div } \vec{f})$ (b) curl $(\text{div } \vec{f})$

(c) grad $(\text{curl } \vec{f})$ (d) grad $(\text{div } \vec{f})$ 1

(vi) A vector $\vec{f}$ is solenoidal if _______.

(a) curl $\vec{f} = 0$ (b) div $\vec{f} = 0$

(c) grad $\vec{f} = 0$ (d) grad $(\text{div } \vec{f}) = 0$ 1

(vii) If the radius of the circle is equal to the radius of the sphere, the circle is called a ______.

(a) small circle (b) imaginary circle

(c) great circle (d) none of these 1

(viii) The equations of the sphere and the plane taken together represent a ______.

(a) sphere (b) plane

(c) straight line (d) circle 1

(ix) Every section of a right circular cone by a plane perpendicular to its axis is ______.

(a) a sphere (b) a cone

(c) a circle (d) a cylinder 1

(x) The general equation of the cone passing through the coordinate axes is ______.

(a) $fyz + gzx + hxy = 0$ (b) $yz + zx + xy = 0$

(c) $ax^2 + by^2 + cz^2 = 0$ (d) $x^2 + y^2 + z^2 = 0$ 1

UNIT—I

2. (a) Show that $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 5

(b) If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, $\vec{b}$ and $\vec{c}$ being non-parallel. 5

3. (p) If $\vec{f}$ is a vector function of t and u is a scalar function of t, then prove that :

$$\frac{d}{dt}(u\vec{f}) = u\frac{d\vec{f}}{dt} + \frac{du}{dt}\vec{f}.$$ 5

(q) Evaluate $\int_1^2 \vec{r} \times \frac{d^2\vec{r}}{dt^2}\, dt$, where

$$\vec{r}(t) = 5t^2\,\vec{i} + t\,\vec{j} - t^3\,\vec{k}.$$ 5

UNIT—II

4. (a) Prove that helices are the only twisted curves whose Darboux's vector has a constant direction. 5

(b) For the curve $x = 3t$, $y = 3t^2$, $z = 2t^3$ at the point $t = 1$, find the equations for osculating plane, normal plane and rectifying plane. 5

5. (p) For the curve $x = a(3t - t^3)$, $y = 3at^2$, $z = a(3t + t^3)$, show that the curvature and torsion are equal. 5

(q) If $\vec{t}' = \vec{d} \times \vec{t}, \vec{n}' = \vec{d} \times \vec{n}, \vec{b}' = \vec{d} \times \vec{b}$, then find the vector $\vec{d}$. 5

6. (a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then show that $\operatorname{div}(r^n \vec{r}) = (n+3)r^n$. 4

(b) Find the directional derivative of $\phi = xy^2 + yz^2$ at the point (2, – 1, 1) in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. 3

(c) If $\phi = 3x^2y - y^3z^2$, find grad ϕ at the point (1, – 2, – 1). 3

7. (p) If $\vec{F} = (2x + y^2)\vec{i} + (3y - 4x)\vec{j}$, evaluate $\int_c \vec{F}.d\vec{r}$ along the parabolic arc $y = x^2$ joining (0, 0) and (1, 1). 5

(q) Apply Green's theorem to prove that the area enclosed by a simple plane curve C is $\frac{1}{2}\int_c (xdy - ydx)$. Hence find the area of an ellipse whose semi-major and semi-minor axes are of lengths a and b. 3+2

UNIT—IV

8. (a) Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 5

(b) Find the equation of the sphere circumscribing the tetrahedron whose faces are :

$x = 0,\ y = 0,\ z = 0,\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$ 5

9. (p) State and prove the condition for the orthogonality of two spheres. 1+4

(q) Find the coordinates of the centre and radius of the circle $x + 2y + 2z = 15$; $x^2 + y^2 + z^2 - 2y - 4z = 11$. 5

UNIT—V

10. (a) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$. 5

(b) Find the equation of right circular cone whose vertical angle is 90° and its axis is along the line $x = -2y = z$. 5

11. (p) Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. 5

(q) Find the equation of the right circular cylinder of radius z whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, – 3, 6. 5