B.Tech I Year I Semester (R19) Regular Examinations January 2020

ALGEBRA & CALCULUS

(Common to all branches)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Find the rank of the matrix $A=\begin{bmatrix}8 & 1 & 3 & 6\\ 0 & 3 & 2 & 2\\ -8 & -1 & -3 & 4\end{bmatrix}$.

(b) If λ is an Eigen value of a matrix A then prove that λ^m is an Eigen value of A^m. (m being a positive integer)

(c) Discuss the application of Rolle's theorem to the function $f(x)=\sec x \ in \ [0,2\pi]$.

(d) State Maclaurin's theorem with Lagrange's form of remainder.

(e) Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $z=\log(x^2+y^2)$.

(f) If $u=x^2-2y^2, v=2x^2-y^2$ find $J=\frac{\partial(u,v)}{\partial(x,y)}$.

(g) Evaluate $\int_0^1\int_0^x e^{\frac{x}{y}}\,dydx$.

(h) Evaluate $\int_0^a\int_0^b\int_0^c (x^2+y^2+z^2)\,dzdydx$.

(i) Show that $\Gamma(n)=2\int_0^\infty e^{-x^2}x^{2n-1}dx,\ (n>0)$.

(j) Express the integral $\int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta$ in terms of beta function.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Reduce the matrix $A=\begin{bmatrix}-2 & -1 & -3 & -1\\ 1 & 2 & 3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$ to Echelon form and hence find its rank.

(b) Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$.

OR

3 (a) Test for consistency the following equations and solve them if consistent :

$$5x+3y+7z=4,$$
$$3x+26y+2z=9,$$
$$7x+2y+10z=5.$$

(b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}$ and hence find its inverse.

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UNIT – II

4 (a) Verify Rolle's theorem for $f(x)=x^{2m-1}(a-x)^{2n}$ in $(0,a)$.

(b) Verify Taylor's theorem for $f(x)=(1-x)^{\frac{5}{2}}$ with Lagrange's form of remainder up to two terms in the interval $[0,1]$.

OR

5 (a) Verify Lagrange's Mean value theorem for $f(x)=(x-1)(x-2)(x-3)$ in $[0,4]$.

(b) Verify Cauchy's mean value theorem for $f(x)=\sin x$ and $g(x)=\cos x$ in the interval $[a,\ b]$.

UNIT – III

6 (a) If $u=x+y+z, uv=y+z, uvw=z$, Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

(b) Discuss the maxima and minima of $f(x,y)=x^3y^2(1-x-y)$.

OR

7 (a) Determine whether the following functions are functionally dependent or not. If functionally dependent, find the functional relation between them: $u=x^2+y^2+2xy+2x+2y,\ v=e^x e^y$.

(b) Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$.

UNIT – IV

8 (a) Change the order of integration and hence evaluate $I=\int_0^a\int_{\sqrt{ax}}^a \frac{y^2\,dy\,dx}{\sqrt{(y^4-a^2x^2)}}$.

(b) Compute the volume of the sphere $x^2+y^2+z^2=a^2$, using spherical coordinates.

OR

9 (a) Evaluate the double integral $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$, using polar coordinates.

(b) Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.

UNIT – V

10 (a) Prove that $\beta\left(m,\frac{1}{2}\right)=2^{2m-1}\beta(m,m)$.

(b) Show that $\int_0^1 x^m(\log x)^n\,dx=\frac{(-1)^n n!}{(m+1)^{n+1}}$, where n is a positive integer and $m>-1$. Hence evaluate $\int_0^1 x(\log x)^3\,dx$.

OR

11 (a) Express the following integrals in terms of gamma functions: (i) $\int_0^\infty \frac{x^c}{c^x}dx$. (ii) $\int_0^\infty a^{-bx^2}dx$.

(b) Express $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of Gamma function and hence evaluate $\int_0^1\frac{dx}{\sqrt{(1-x^n)}}$.

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