Code: 13A54101 R13

B.Tech I Year (R13) Regular Examinations June/July 2014

MATHEMATICS - I

(Common to all branches)

Time: 3 hours Max. Marks: 70

Part – A
(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 M)

(a) Solve $\frac{d^2y}{dx^2} + 1.5\frac{dy}{dx} + 0.5y = 0$.

(b) Solve $(e^y + 1)\cos x dx + e^y \sin x dy = 0$.

(c) Find Taylor's series expansion for $\tan^{-1}\left(\frac{y}{x}\right)$ about $(1, 1)$.

(d) Find the radius of the curvature at the origin for the curve $2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0$.

(e) Find the asymptote of $y = \frac{x^2 + 2x - 1}{x}$.

(f) Evaluate $\int_0^1 \int_0^x e^{x+y}\, dydx$.

(g) Find $L\{Cos^2 t\}$.

(h) Find $L^{-1}\left\{\frac{e^{-3s}}{s+2}\right\}$.

(i) Show that $\nabla.(r^n \bar{r}) = (n+3)r^n$.

(j) State Stokes theorem.

Part – B
Answer all five units (5 X 10 = 50 M)

UNIT - I

2 A mass m suspended from one end of a spring is subjected to a periodic force $f = f_0 \sin at$ in the direction of its length. The force f is measured positive vertically downwards and at time t = 0, m is at rest. If the spring constant is K, prove that the displacement of m at time t is given by $x = \frac{f_0}{m(p^2 - a^2)}\left(\sin at - \frac{a}{p}\sin pt\right)$ where $p^2 = {}^{K}/_{m}$. Neglect the damping effects.

OR

3 Solve $(x^2D^2 + xD + 1)y = \log x \sin(\log x)$.

UNIT - II

4 Discuss the maxima and minima of $f(x, y) = \sin x \sin y \sin(x + y)$.

OR

5 Prove that the evolute of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ is another cycloid.

UNIT - III

6 Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the straight line y = x.

OR

7 Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dz\, dx\, dy$.

UNIT - IV

8 Using convolution theorem solve the IVP:
$y''(t) + 3y'(t) + 2y(t) = e^{-t}, \quad y(0) = 0, \quad y'(0) = -1$

OR

9 Find $L^{-1}\left\{\frac{s(a^2 - b^2)}{s^4(a^2 + b^2)s^2 + a^2b^2}\right\}$.

UNIT - V

10 For a solenoidal vector $\bar{f}$, prove that $\nabla x(\nabla x(\nabla x(\nabla x\bar{f}))) = \nabla^4\bar{f}$.

OR

11 Evaluate $\int_C [(2xy^3 - y^2\cos x)dx + (1 - 2y\sin x + 3x^2y^2)dy]$ where C is the arc of the parabola $2x = \pi y^2$ from (0, 0) to $\left(\frac{\pi}{2}, 1\right)$.

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