

Code: 13A54101

R13

B.Tech I Year (R13) Regular Examinations June/July 2014

MATHEMATICS - I

(Common to all branches)

Time: 3 hours

Max. Marks: 70

Part – A (Compulsory Question)

1 Answer the following: (10 X 02 = 20 M)

- Solve $\frac{d^2y}{dx^2} + 1.5\frac{dy}{dx} + 0.5y = 0$.
- Solve $(e^y + 1) \cos x dx + e^y \sin x dy = 0$.
- Find Taylor's series expansion for $\tan^{-1}\left(\frac{y}{x}\right)$ about $(1, 1)$.
- Find the radius of the curvature at the origin for the curve $2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0$.
- Find the asymptote of $y = \frac{x^2 + 2x - 1}{x}$.
- Evaluate $\int_0^1 \int_0^x e^{x+y} dy dx$.
- Find $L\{\cos^2 t\}$.
- Find $L^{-1}\left\{\frac{e^{-3s}}{s+2}\right\}$.
- Show that $\nabla \cdot (r^n \vec{r}) = (n+3)r^n$.
- State Stokes theorem.

Part – B

Answer all five units (5 X 10 = 50 M)

UNIT - I

- A mass m suspended from one end of a spring is subjected to a periodic force $f = f_0 \sin at$ in the direction of its length. The force f is measured positive vertically downwards and at time $t = 0$, m is at rest. If the spring constant is K , prove that the displacement of m at time t is given by $x = \frac{f_0}{m(p^2 - a^2)} \left(\sin at - \frac{a}{p} \sin pt \right)$ where $p^2 = K/m$. Neglect the damping effects.

OR

- Solve $(x^2 D^2 + xD + 1)y = \log x \sin(\log x)$.

UNIT - II

- Discuss the maxima and minima of $f(x, y) = \sin x \sin y \sin(x + y)$.

OR

- Prove that the evolute of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ is another cycloid.

UNIT - III

- Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the straight line $y = x$.

OR

- Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z dz dx dy$.

UNIT - IV

- Using convolution theorem solve the IVP:
 $y''(t) + 3y'(t) + 2y(t) = e^{-t}$, $y(0) = 0$, $y'(0) = -1$

OR

- Find $L^{-1}\left\{\frac{s(a^2 - b^2)}{s^4(a^2 + b^2)s^2 + a^2b^2}\right\}$.

UNIT - V

- For a solenoidal vector \vec{f} , prove that $\nabla \times (\nabla \times (\nabla \times (\nabla \times \vec{f}))) = \nabla^4 \vec{f}$.

OR

- Evaluate $\int_C [(2xy^3 - y^2 \cos x)dx + (1 - 2ysinx + 3x^2y^2)dy]$ where C is the arc of the parabola $2x = \pi y^2$ from $(0, 0)$ to $(\frac{\pi}{2}, 1)$.
