Code: 13A54101 **R13**

B.Tech I Year (R13) Supplementary Examinations December 2019

MATHEMATICS – I

(Common to all branches)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Solve $(x^2 - y^2)dx = 2xydy$.

(b) Solve the differential equation : $(D^4 - 2D^3 + 2D^2 - 2D + 1)y = 0$

(c) Solve the differential equation: $(x^2D^2 - 3xD + 1)y = 0$.

(d) If $U = \log(x^3 + y^3 + z^3 - 3xyz)$ then find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

(e) If $x = r\cos\theta,\ y = r\sin\theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$?

(f) If $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ then find critical points.

(g) Evaluate $\int_0^1\int_0^1 \frac{1}{\sqrt{(1-x^2)(1-y^2)}}dxdy$.

(h) Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\, dxdydz$.

(i) Find $Curl\ \bar{F}$ for $\bar{F} = z\bar{i} + x\bar{j} + y\bar{k}$.

(j) State Gauss divergence theorem.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Solve $(D^4 + 2D^2 + 1)y = e^x \cos x$.

(b) Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter.

OR

3 (a) Solve $(D^2 - 2D + 1)y = x\, e^x \sin x$.

(b) Using method of variation of parameter Solve $(D^2 + 4)y = \tan 2x$.

UNIT – II

4 (a) Verify Rolle's theorem for the function f(x) = $\frac{\sin x}{e^x}$ $in\ [0, \pi]$.

(b) Find the coordinates of the centre of curvature at $(at^2,\ 2at)$ on the parabola $y^2 = 4ax$.

OR

5 (a) Find the maximum and minimum values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

(b) Find C of Cauchy's Mean Value theorem for $f(x) = \sin x,\ g(x) = \cos x\ in\ a, b$.

Contd. in page 2

UNIT – III

6 (a) Trace the curve $y^2 (a - x) = x^2 (a + x)$.

(b) Find the surface area of the solid of revolution of one loop of the curve $r^2 = a^2 \cos 2\theta$ about the initial line.

OR

7 (a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$.

(b) By changing the order of integration , evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$.

UNIT – IV

8 (a) Find the Laplace transform of: (i) $\left\{\frac{\sin 3t.\cos t}{t}\right\}$. (ii) $\{t^2 \sin 2t\}$.

(b) Apply Convolution theorem to find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.

OR

9 (a) Find the Laplace transform of e^{-3t} (2 cos 5t – 3 sin 5t).

(b) Solve the D.E $y'' - 2y' - 8y = 0,\ y(0) = 3,\ y'(0) = 6$. Using Laplace transform.

UNIT – V

10 (a) Find the directional derivative of $xyz^2 + xz$ at (1, 1, 1) in the direction of $\bar{i} + 2\bar{j} + 3\bar{k}$.

(b) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2\,dy]$ where C is bounded by $y = x$ and $y = x^2$.

OR

11 Verify Gauss divergence theorem for $\bar{F} = 4xy\bar{i} - y^2\bar{j} + 4z\bar{k}$ where S is the surface of the cube bounded by x = 0, x = 1; y = 0, y = 1; z = 0, z = 1.

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